
Iterative Learning of Feed-Forward Corrections for High-Performance Tracking

Fabian L. Mueller, Angela P. Schoellig, Raffaello D'Andrea

Institute for Dynamic Systems and Control

ETH Zürich, Switzerland



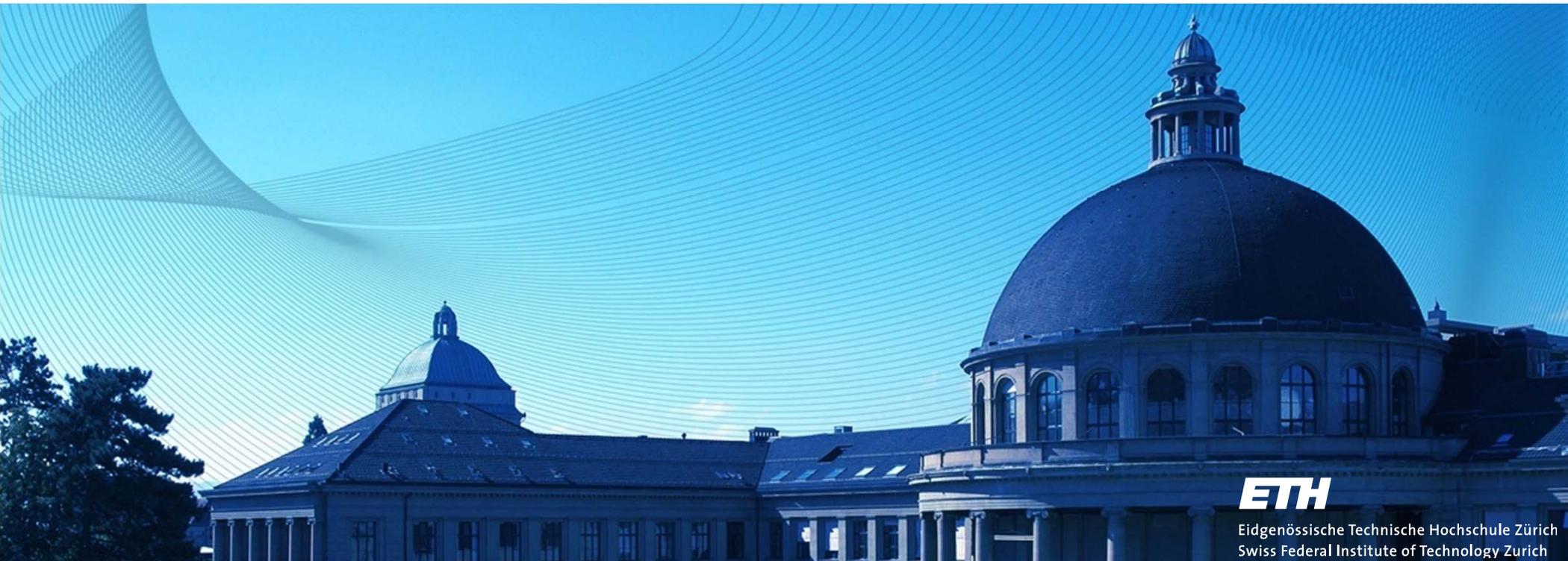
ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Iterative Learning of Feed-Forward Corrections for High-Performance Tracking

Fabian L. Mueller, Angela P. Schoellig, Raffaello D'Andrea

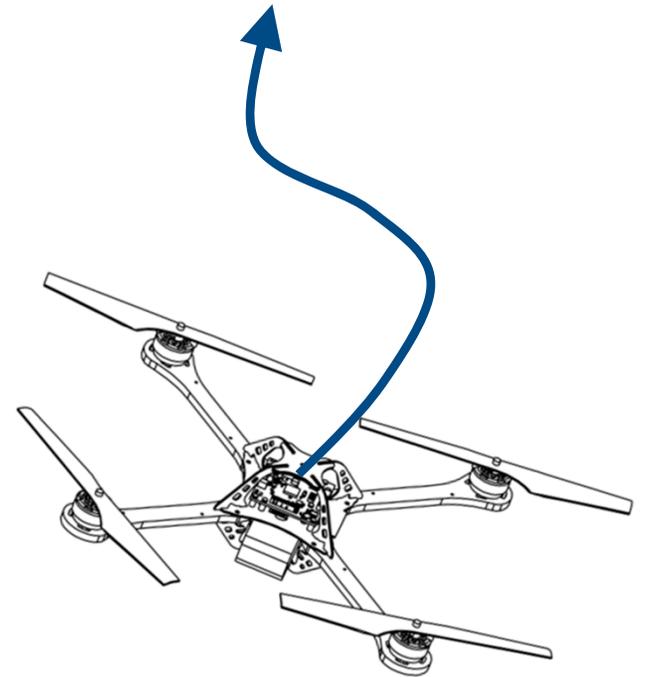
Institute for Dynamic Systems and Control
ETH Zürich, Switzerland



ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

GOAL – Precise tracking of a desired output trajectory



GOAL – Precise tracking of a desired output trajectory

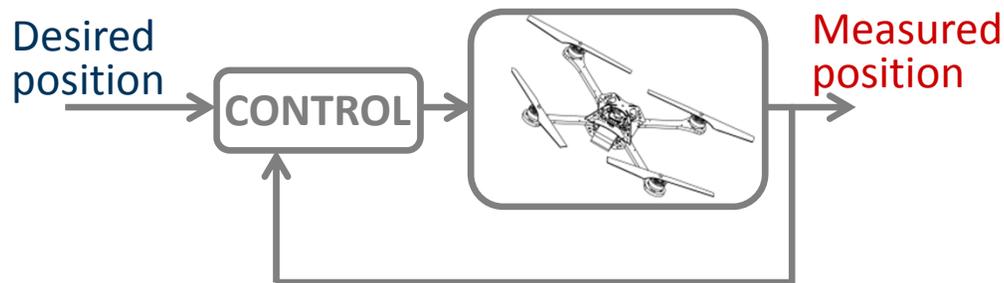
Example: Quadrotor vehicle



GOAL – Precise tracking of a desired output trajectory

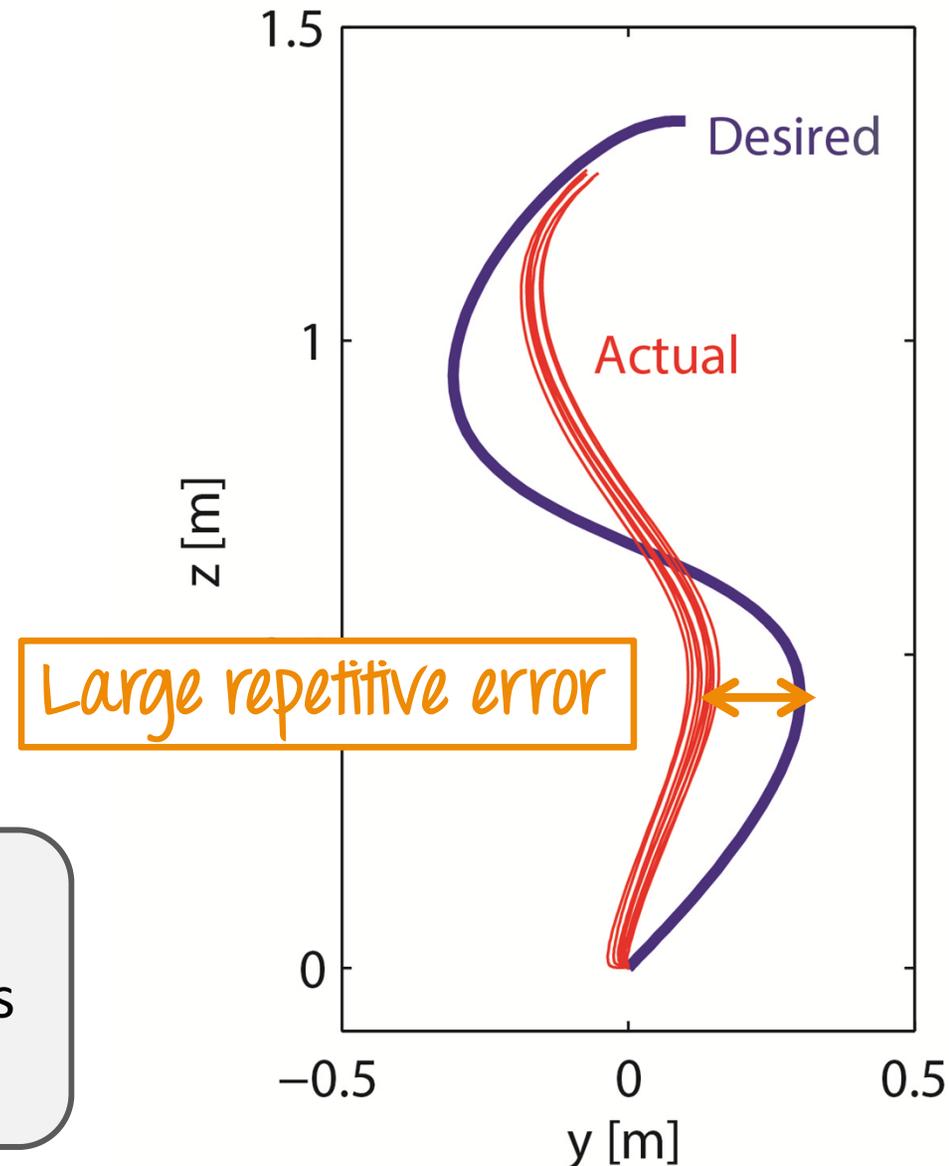
Example: Quadrotor vehicle

Typical setup: Feedback control



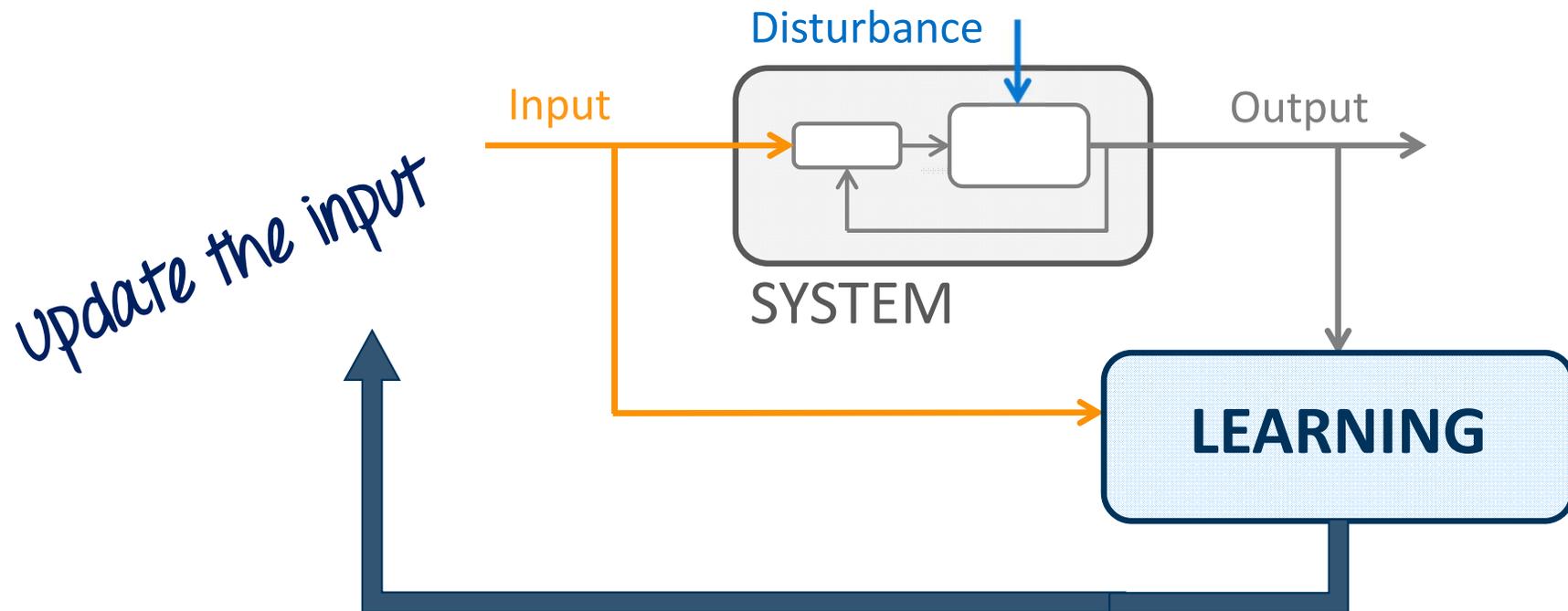
Limitations of feedback control:

Disturbances and unmodelled dynamics
(non-zero mean)



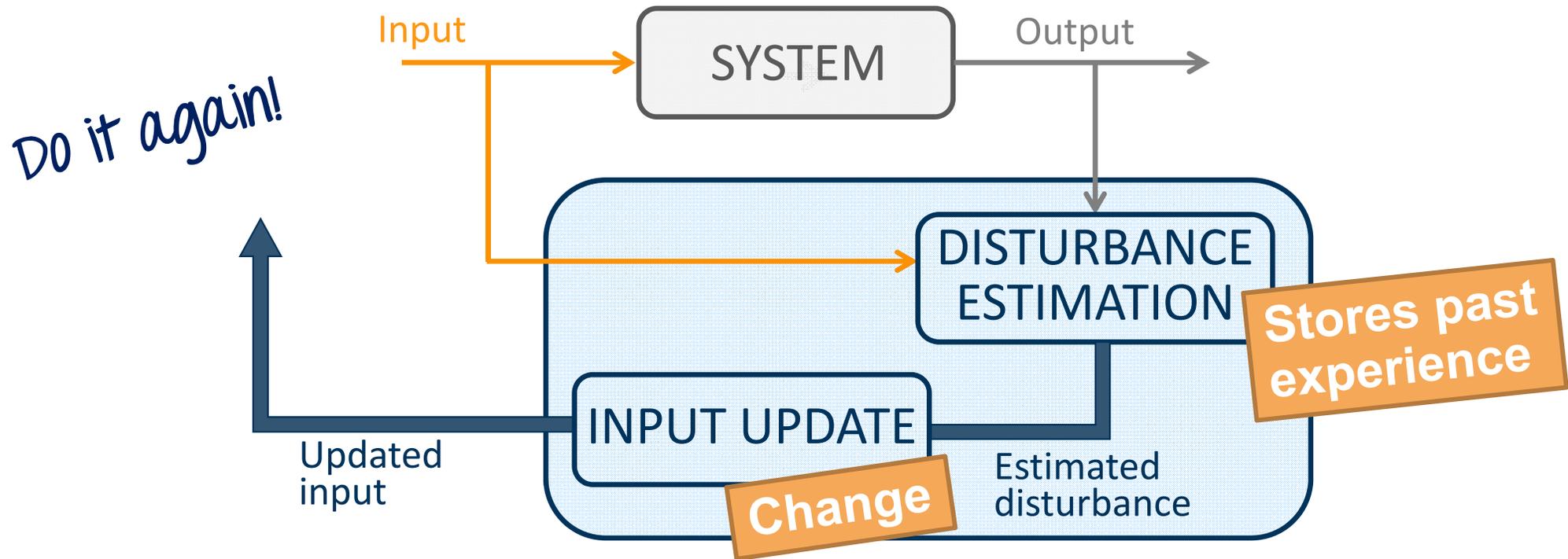
LEARNING APPROACH

Improve the performance over causal, feedback control by learning from a repeated operation.



Potential: Acausal action, anticipating repetitive disturbances.

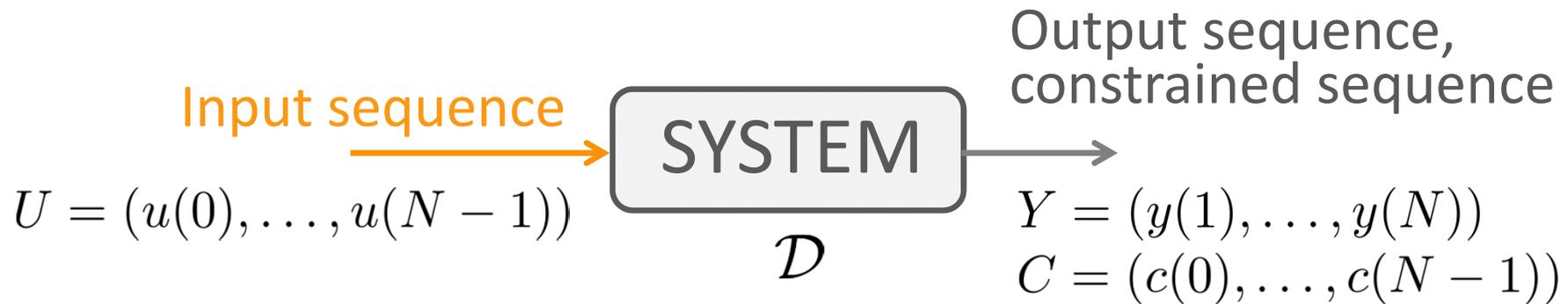
LEARNING APPROACH



1. Dynamics model (here: from numerical simulation)
2. Disturbance estimation*
3. Update of input trajectory*

* Angela P. Schoellig, Fabian L. Mueller, Raffaello D'Andrea, "Optimization-based iterative learning for precise quadcopter trajectory tracking," Autonomous Robots, 2012

1 | DYNAMICS MODEL



Prerequisites:

- **Coarse model** $\mathcal{D} : U \rightarrow (Y, C)$
- **Desired output trajectory** Y^*
with corresponding nominal input $(Y^*, C^*) = \mathcal{D}(U^*)$

1 | DYNAMICS MODEL

Define:

- **Linear mapping** from *input deviations* to *changes in output and constrained variables*:

$$y = F u, \quad c = L u$$

$$u = U - U^*, \\ y = Y - Y^*, \quad c = C - C^*$$

From numerical dynamics simulation:

- Obtain F, L by running $2N$ identification runs
 - Apply $U_{i+} = (u^*(0), \dots, u^*(i) + \Delta u, \dots, u^*(N-1)) \rightarrow (Y_{i+}, C_{i+})$
 - Obtain $F(:, i) = (Y_{i+} - Y^*) / \Delta u$
 $L(:, i) = (C_{i+} - C^*) / \Delta u$ ← **ith column**

1 | ITERATION-DOMAIN MODEL

For each trial j , $j \in \{1, 2, \dots\}$,

$$y_j = F u_j + d_j + \mu_j.$$

Recurring disturbance d_j .

Unknown. Only small changes between iterations:

$$d_{j+1} = d_j + \omega_j.$$

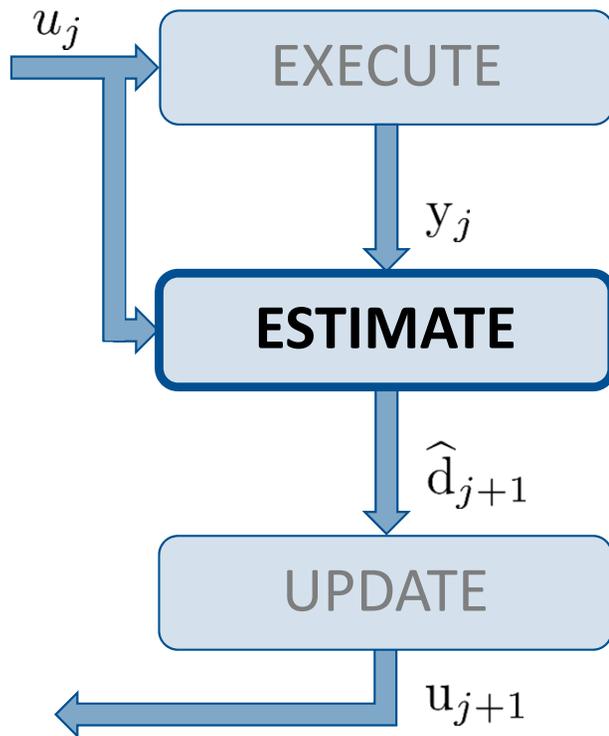
μ_j, ω_j — trial-uncorrelated,
zero-mean Gaussian
noise

Noise μ_j .

Unknown. Changing from iteration to iteration.

From trial to trial our knowledge about d_j improves.

ESTIMATION



UPDATE OF DISTURBANCE ESTIMATE
via **Kalman filter** in the iteration domain:

Prediction step:

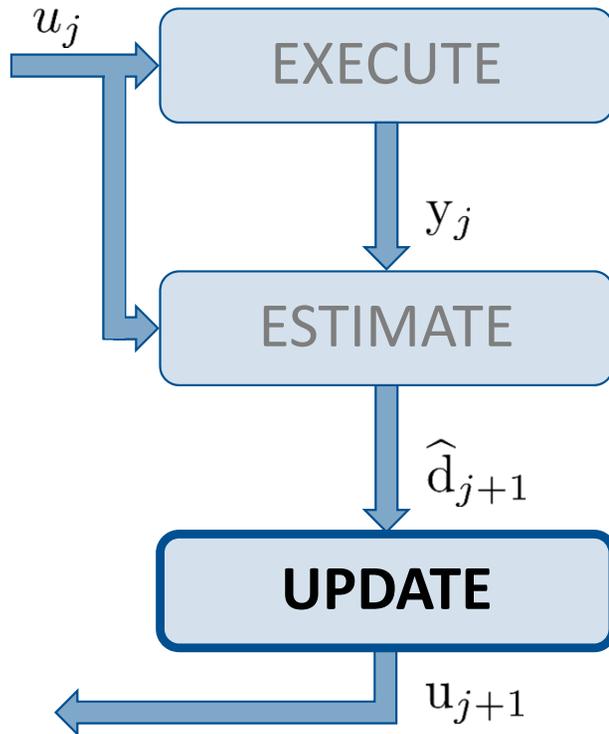
$$d_{j+1} = d_j + \omega_j.$$

Measurement update step:

$$y_j = F u_j + d_j + \mu_j.$$

➔ Obtain \hat{d}_{j+1} .

INPUT UPDATE



INPUT UPDATE via **convex optimization**:

minimizes the expected tracking error in the next trial:

$$\mathbb{E}[y_{j+1} | \text{all past measurements}] = F u_{j+1} + \hat{d}_{j+1}.$$

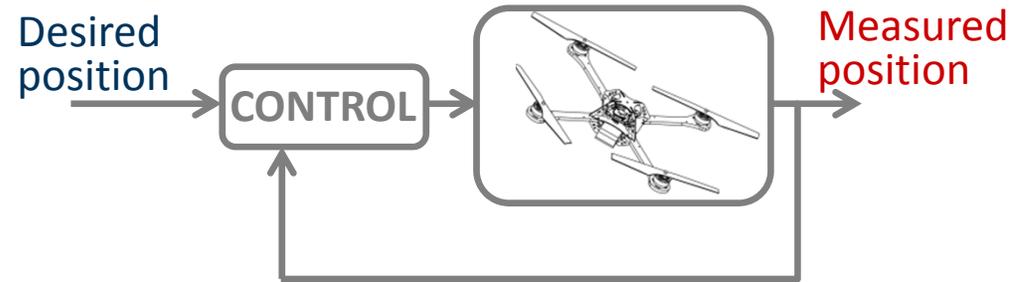
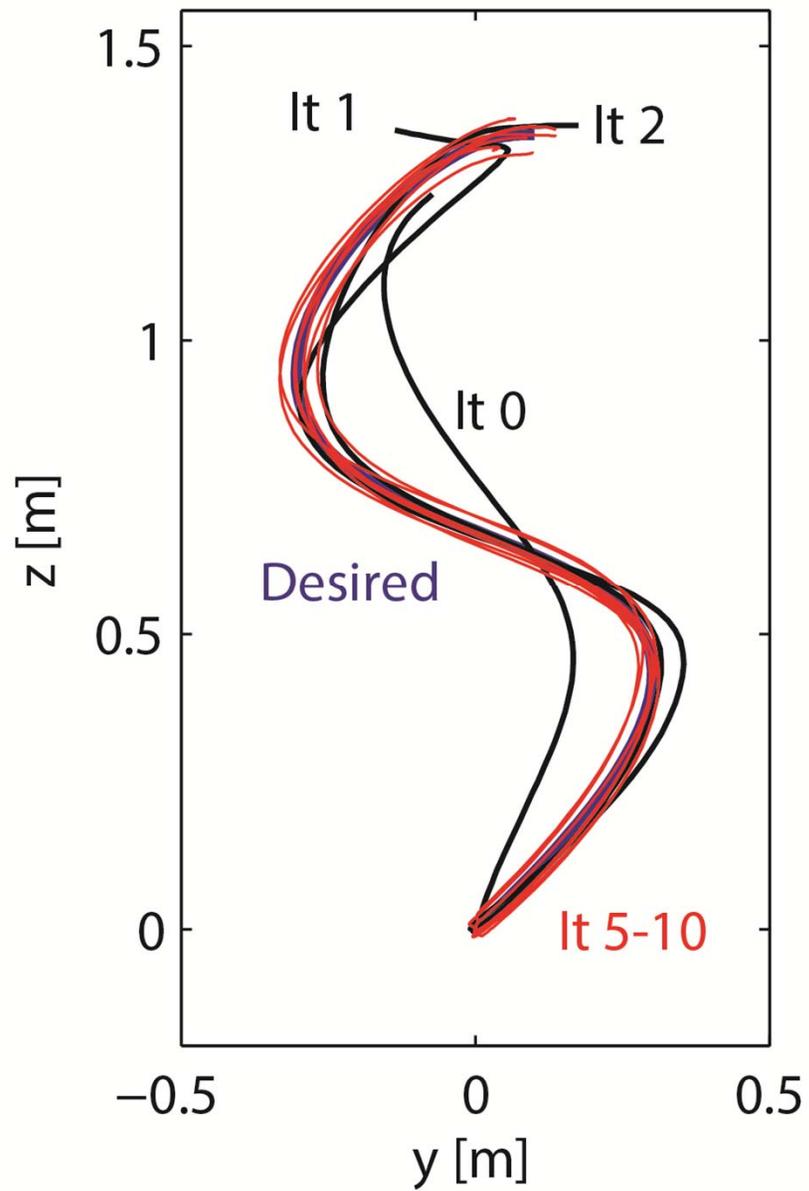
$$\min_{u_{j+1}} \left\| F u_{j+1} + \hat{d}_{j+1} \right\|_p \quad p \in \{1, 2, \infty\}$$

subject to

$$L u_{j+1} \preceq c_{\max}$$

➔ Obtain u_{j+1} .

EXPERIMENTAL RESULTS



VIDEO: <https://youtu.be/zHTCsSkmADo?list=PLC12E387419CEAFF2>

Quadrocopter Slalom Learning

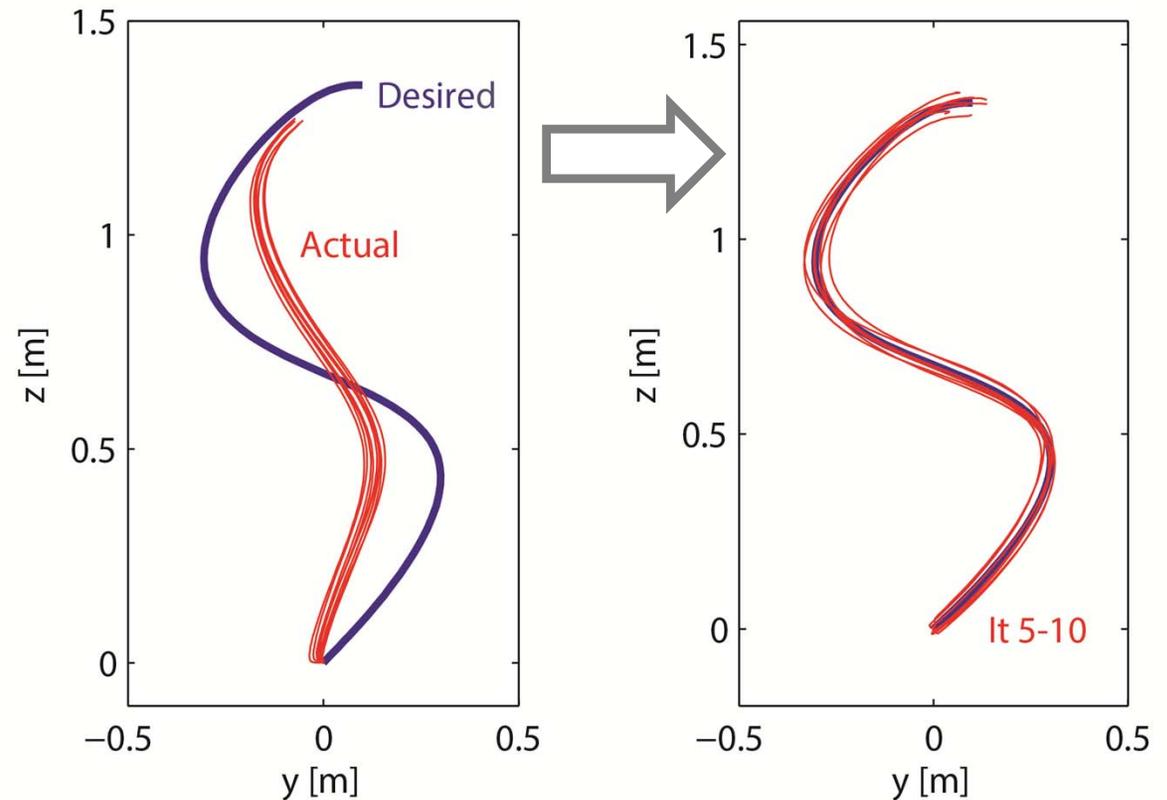


ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

CONCLUSIONS

- Learning algorithm combines **model data** with **experimental data**
- Convergence in around 5-10 iterations



Repetitive error components can be effectively compensated for by learning from past data.

Result is an **improved tracking performance**.