Design of Norm-Optimal Iterative Learning Controllers: The Effect of an Iteration-Domain Kalman Filter for Disturbance Estimation

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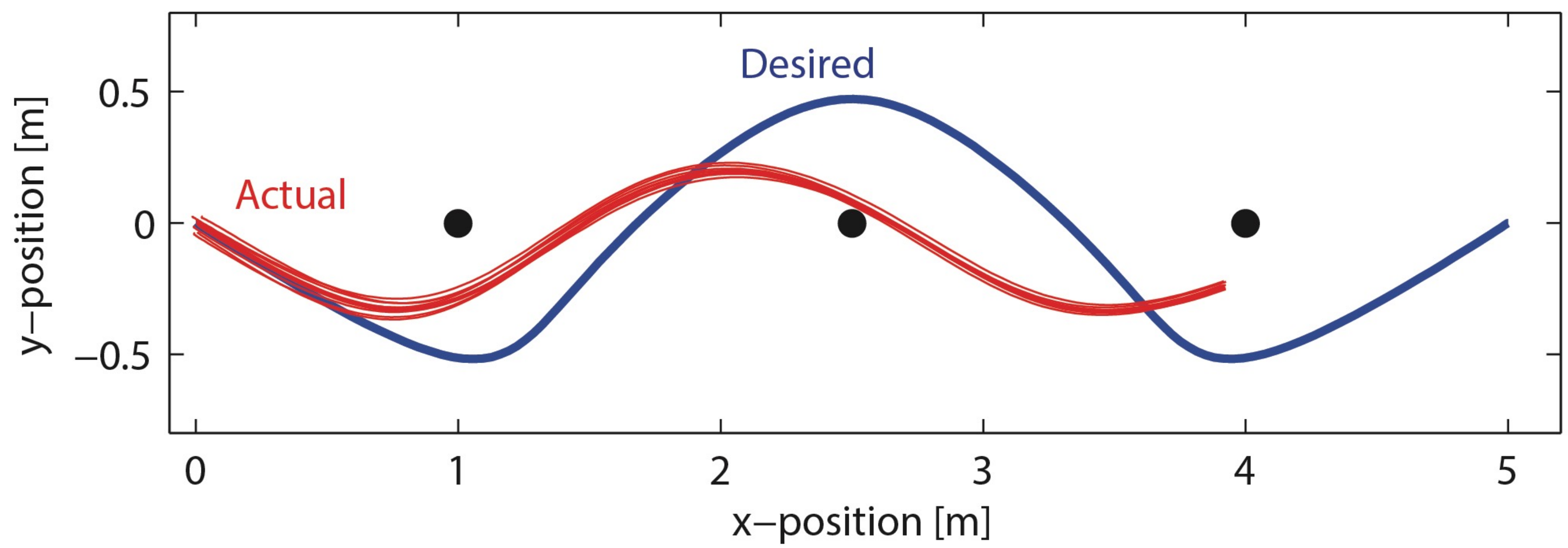
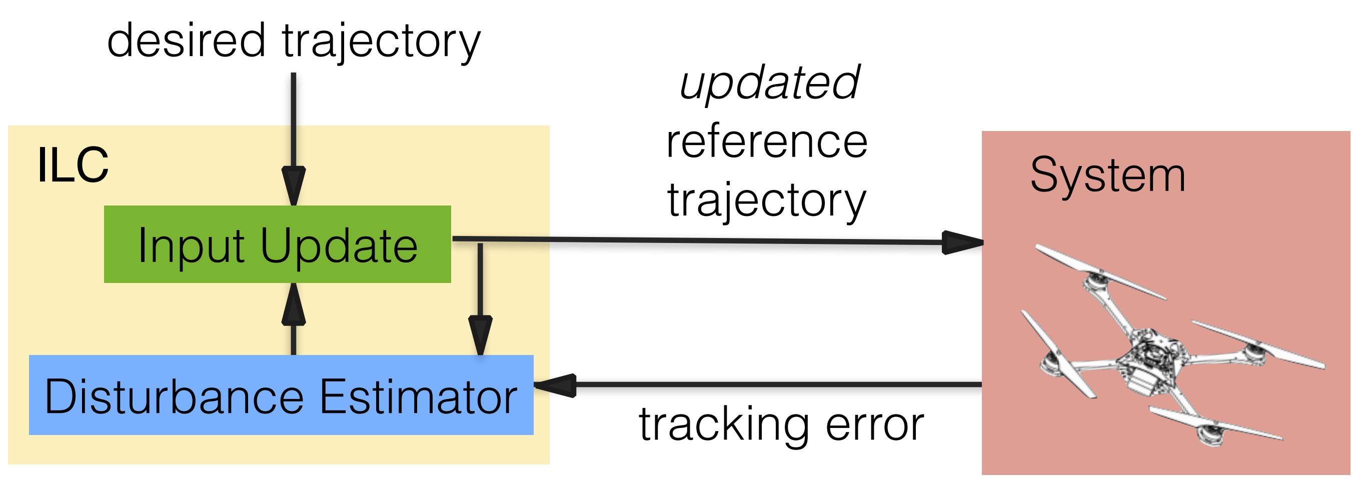
Quadrocopter Tracking Performance

Problem: Unsatisfactory tracking performance



Solution: Iterative Learning Control with Kalman Filter (K-ILC)



A. P. Schoellig, F. L. Mueller, and R. D'Andrea, "Optimization-based iterative learning for precise quadrocopter trajectory tracking," Autonomous Robots, vol. 33, no. 1-2, pp. 103–127, 2012.

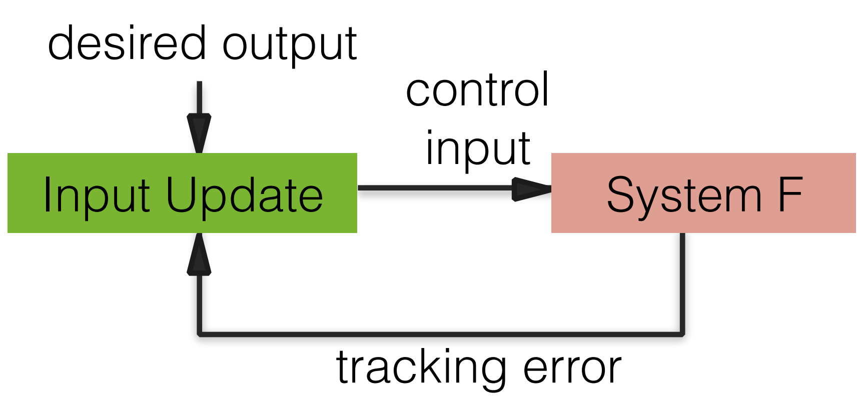
F. L. Mueller, A. P. Schoellig, and R. D'Andrea, "Iterative learning of feed-forward corrections for high-performance tracking," in Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2012, pp. 3276–3281.

Goal: Analytic Comparison of ILC Algorithms

Compare QILC and K-ILC
What are the differences?

QILC

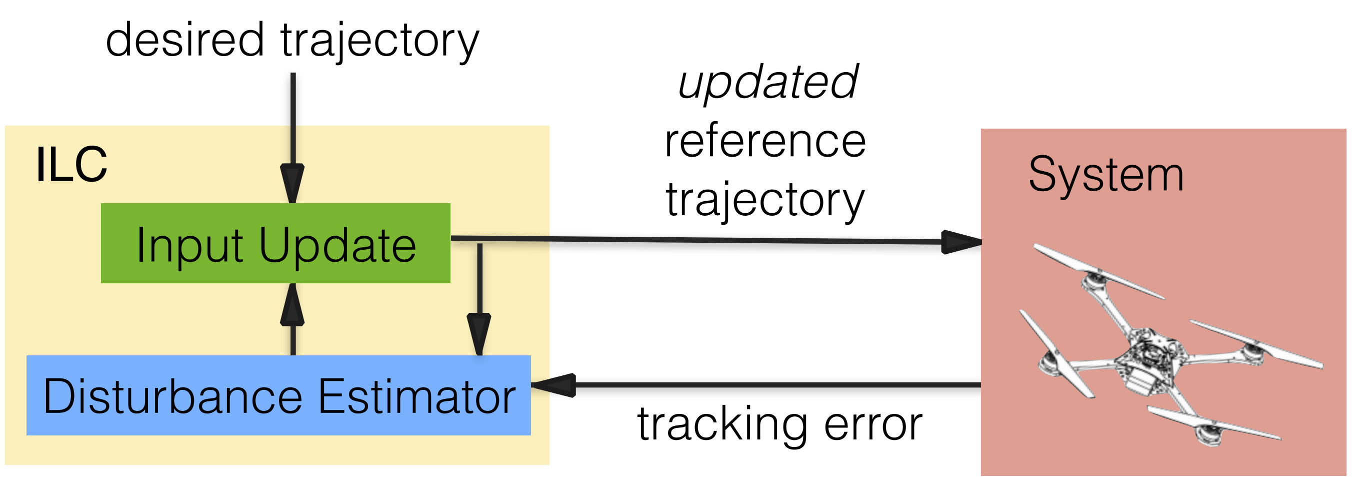
- **Q**uadratic cost criterion **ILC**



J. H. Lee, K. S. Lee, and W. C. Kim, “Model-based iterative learning control with a quadratic criterion for time-varying linear systems,” Automatica, vol. 36, pp. 641–657, 2000.

K-ILC

- **K**alman-Filter-Enhanced **ILC**



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Outline of the Presentation

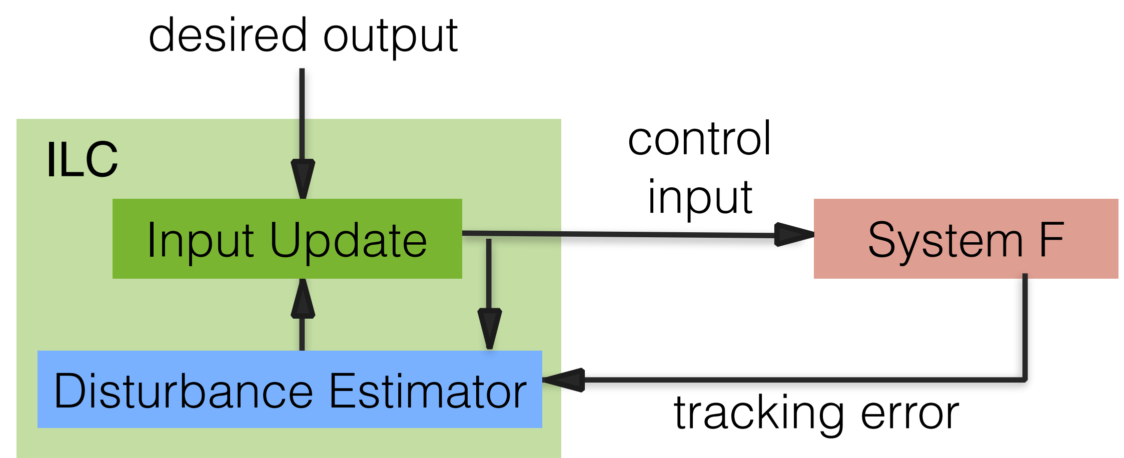
1. Detailed Presentation of **K-ILC Algorithm**
2. **Comparison** with Standard QILC
3. **Simulation** Example

Lifted-Domain Representation

Lifted vector notation for **j-th** Iteration:

$$u_j = [u_j[1], u_j[2], ..., u_j[N]]^T$$

Equivalent for all other signals



Nominal System Model:
Linear, Discrete, Iteration-Constant

$$y_j = \overbrace{\begin{bmatrix} CB & 0 & 0 & 0 \\ CAB & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ CA^{N-1}B & \dots & CAB & CB \end{bmatrix}}^{F} u_j$$

Time-constant linear system for illustration

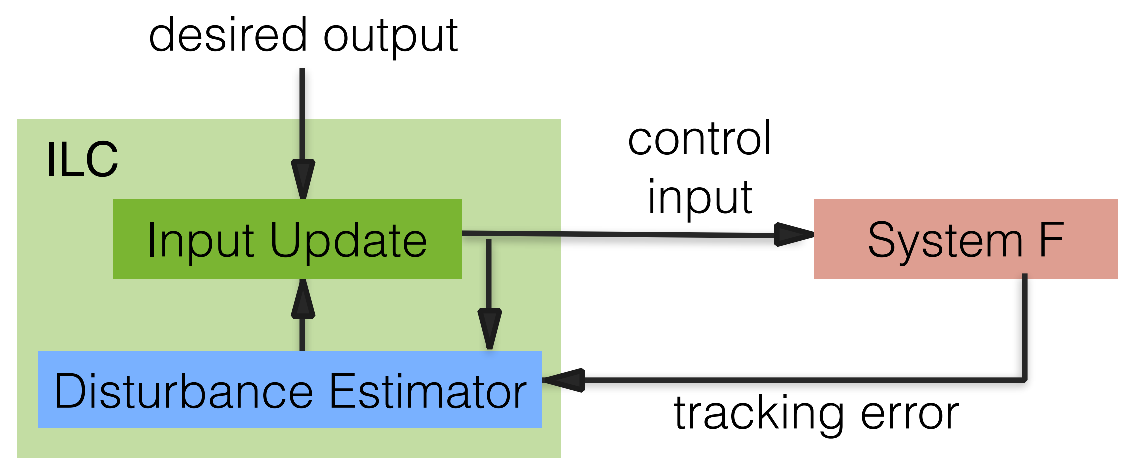
Measured tracking error:

$$e_j = y_j - y_{desired}$$

Disturbance Estimation of K-ILC Algorithm

Linearised system F around desired trajectory:

$$y_j = F u_j$$



System Model Including Modelled **Disturbance** as **Stochastic Process**:

$$\boxed{\begin{aligned} d_{j+1} &= d_j + \omega_j \\ y_j &= F u_j + d_j + \mu_j \end{aligned}}$$

stochastic disturbance representing modelling errors

modelled system output

$$\omega_j \sim \mathcal{N}(0, E_j), \mu_j \sim \mathcal{N}(0, H_j)$$
$$d_0 \sim \mathcal{N}(0, P_0)$$

random variable distributions

Kalman filter equations:
$$\begin{aligned} S_j &= P_j + E_j \\ K_j &= S_j (S_j + H_{j+1})^{-1} \\ P_j &= (I - K_j) S_j . \end{aligned}$$

iteration-varying Kalman gain K_j

Input Update of K-ILC Algorithm

A **Error prediction** of next iteration:

$$\bar{e}_{j+1} = \underbrace{F u_{j+1} - y_d}_{\text{nominal model error}} + \hat{d}_{j+1}$$

Kalman filter used through Estimation of Disturbance:

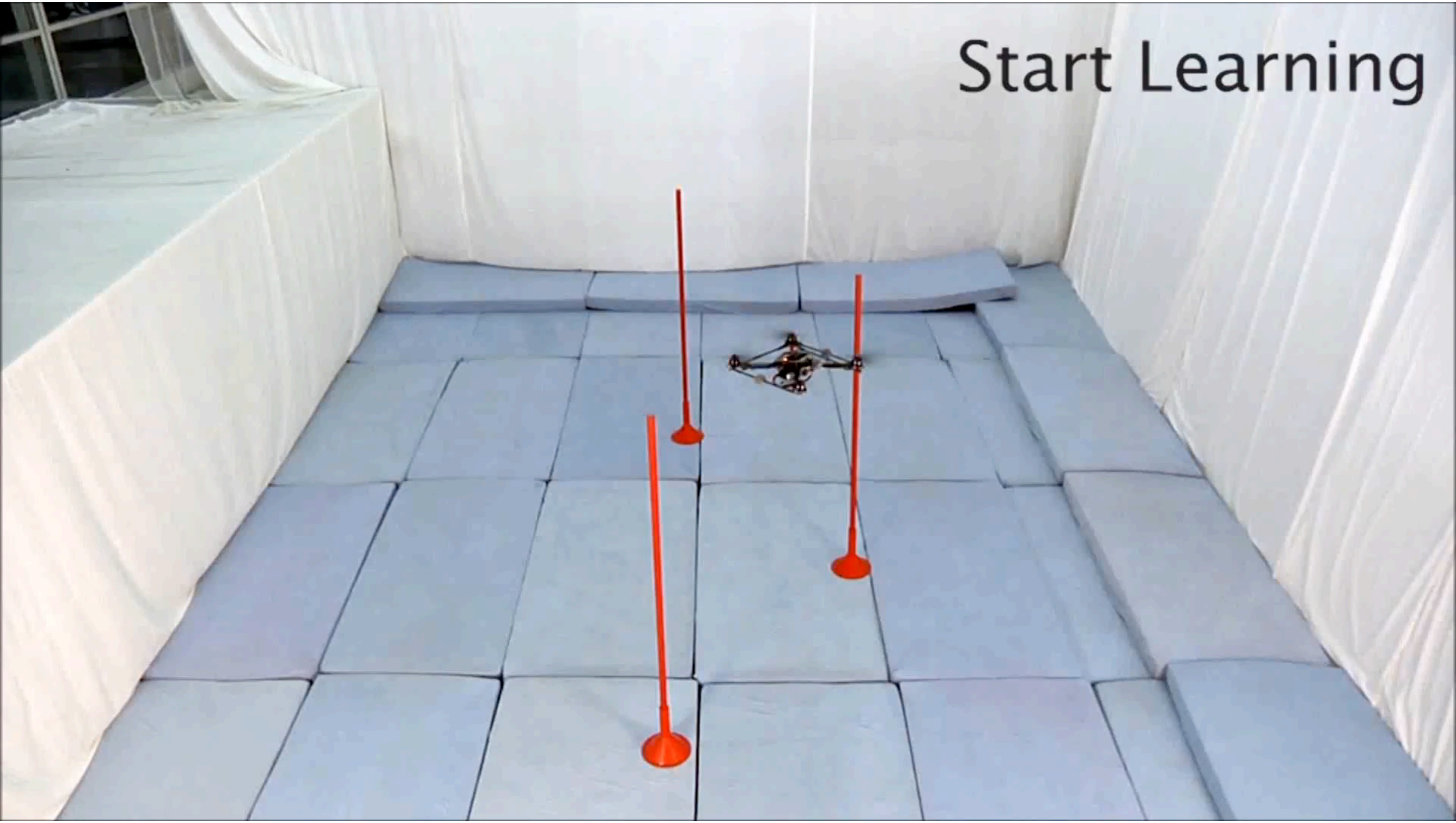
$$\hat{d}_{j+1} = \hat{d}_j + K_j (y_d - F u_j - \hat{d}_j)$$

B Updated **input** as solution of **convex optimisation** of cost function:

$$u_{j+1} = \underset{u'_{j+1} \in C}{\operatorname{argmin}} \{ J_{j+1}(u'_{j+1}) \}$$

$$J_{j+1} = \bar{e}_{j+1}^T W_e \bar{e}_{j+1}$$

Video of ILC in Action



Goal: Analytic Comparison of ILC Algorithms

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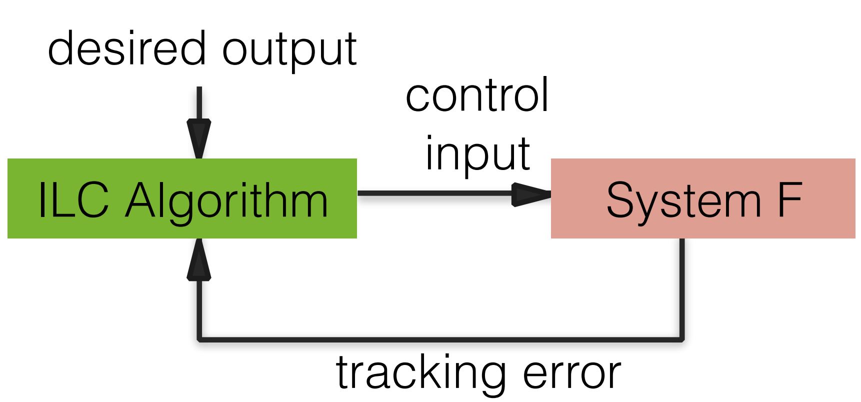
Objective: Compare QILC and K-ILC

QILC

- **Q**uadratic cost criterion **ILC**
- Deterministic system model



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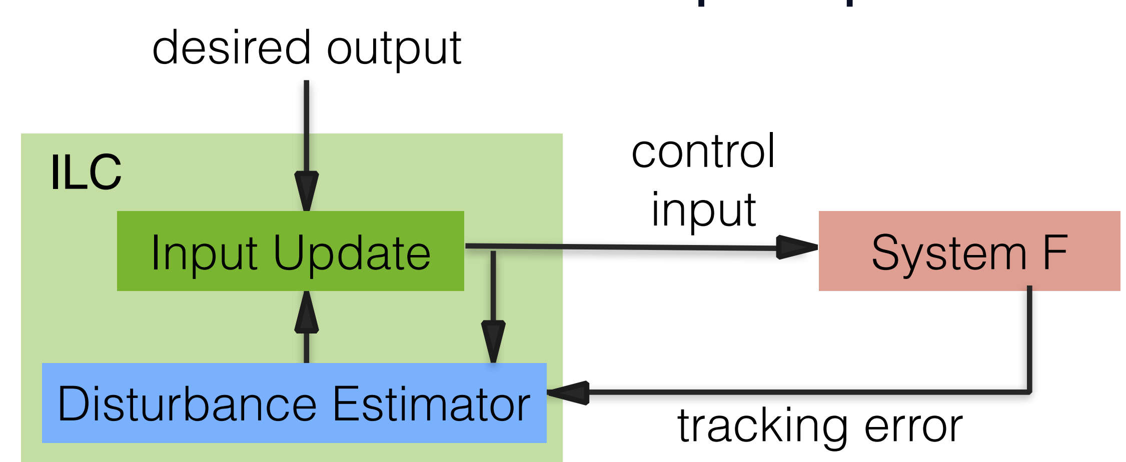
K-ILC

- **K**alman-Filter**-**Enhanced **ILC**
- Modelling errors as stochastic disturbance
- Separated disturbance estimation and input update



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Comparison of Input Update

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QILC	K-ILC

A Error prediction

QILC

$$\bar{e}_{j+1} = F\Delta u_{j+1} + e_j$$

➤ $\Delta u_{j+1} = u_{j+1} - u_j$

K-ILC

$$\bar{e}_{j+1} = \underbrace{F u_{j+1} - y_d}_{\text{nominal model error}} + \hat{d}_{j+1}$$

noise filtering

B Input update cost function

QILC

$$J_{j+1} = \bar{e}_{j+1}^T W_e \bar{e}_{j+1} + \Delta u_{j+1}^T W_{\Delta u} \Delta u_{j+1}$$

K-ILC

$$J_{j+1} = \bar{e}_{j+1}^T W_e \bar{e}_{j+1}$$

Parameters Defining the Algorithms

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QILC | K-ILC

Parameters

$$J_{j+1} = \bar{e}^T_{j+1} \underline{W_e} \bar{e}_{j+1} + \Delta u^T_{j+1} \underline{W_{\Delta u}} \Delta u_{j+1}$$

noise filtering

$$d_{j+1} = d_f + \omega_j$$
$$y_j = F u_j + d_j + \mu_j,$$

$$\omega_j \sim \mathcal{N}(0, \underline{E_j}), \mu_j \sim \mathcal{N}(0, \underline{H_j})$$

$$d_0 \sim \mathcal{N}(0, \underline{P_0})$$

$$\begin{aligned} S_j &= P_j + E_j \\ \underline{K_j} &= S_j (S_j + H_{j+1})^{-1} \\ P_j &= (I - K_j) S_j . \end{aligned}$$

2 Weighting Matrices

3 Covariance Matrices

Quadratic Norm Allows an Explicit Comparison

$$^{QILC}u_{j+1} = u_{nom} - \sum_{i=1}^{j} {}^{QILC}L e_j$$

$$^{K-ILC}u_{j+1} = u_{nom} - \sum_{i=1}^{j} {}^{K-ILC}L_j e_j$$

Explicit notation possible with **quadratic norm** and **no constraints**!

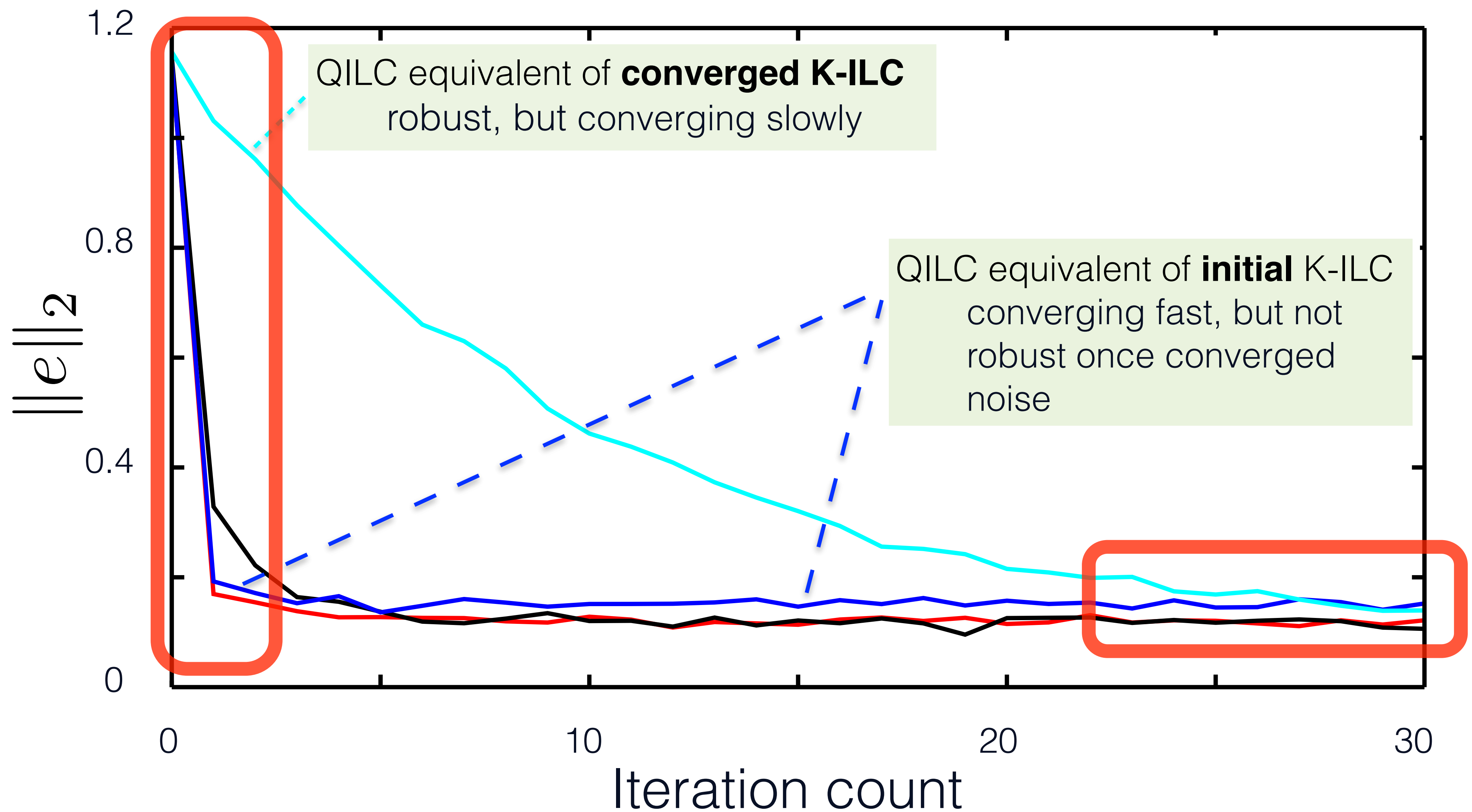
$$^{QILC}L = (\overset{0}{\cancel{W_{\Delta u}}} + F^{\mathrm{T}} W_e F)^{-1} F^{\mathrm{T}} W_e \; \boxed{= F^{-1}}$$

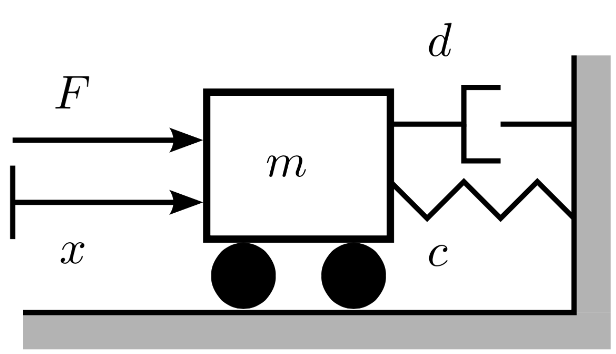
$$^{K-ILC}L_j = F^{-1} \overset{I}{\cancel{K_j}} \quad \boxed{= F^{-1}}$$

For given iteration QILC can be made **equivalent** to K-ILC

➤K-ILC **optimises** gain for **every iteration**

Mass-Spring-Damper Simulation Example





K-ILC designed for the problem
QILC designed for the problem
QILC equivalent of converged K-ILC
QILC equivalent of initial K-ILC

Advantages of K-ILC Algorithm

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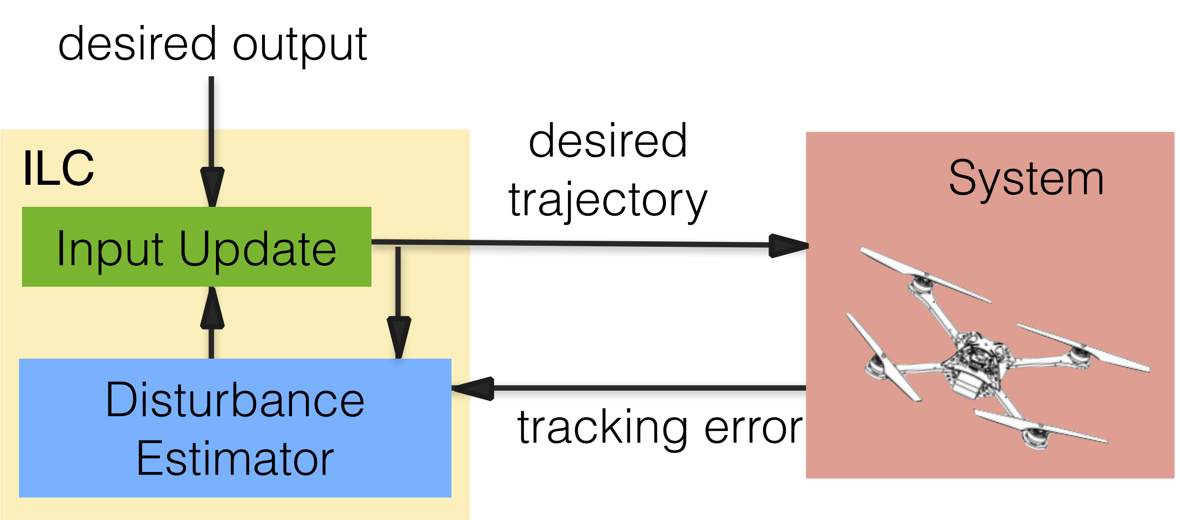
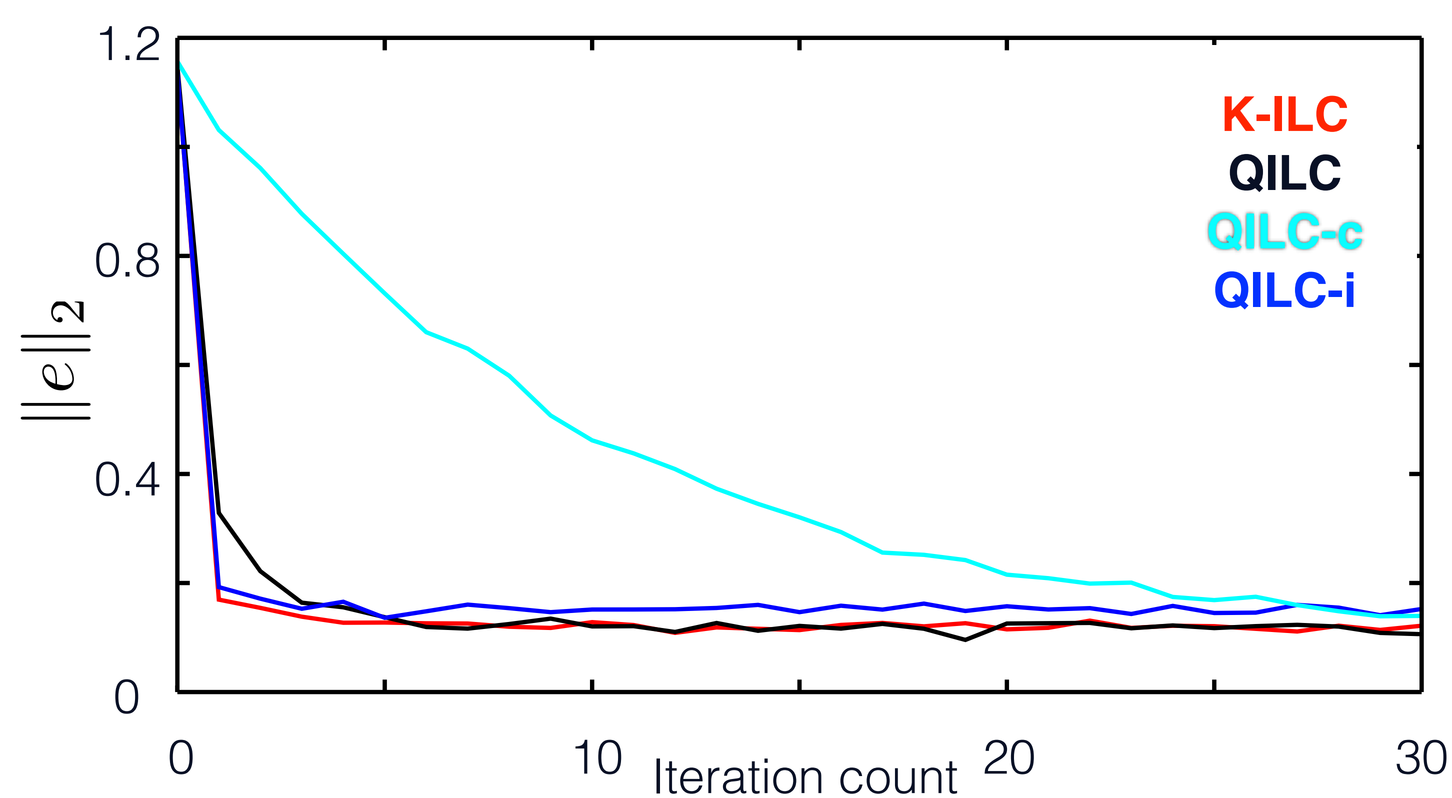
Implications of Kalman filter usage:

1. **Separation** between disturbance estimation and input update
2. Straightforward **iteration-varying and optimal** input update behaviour:
    - Fast initial convergence behaviour
    - Noise-resilient converged behaviour





Thank you!

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