Chapter 4
So You Think You Can Dance? Rhythmic Flight Performances with Quadrocopters

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4.1 Rhythmic Flight with Quadrocopters

Fly with the music. —Song title by DJ Grande

This chapter presents a set of algorithms that enable quadrotor vehicles (such as the ones depicted in Fig. 4.1) to "fly with the music"; that is, to perform rhythmic motions that are aligned with the beat of a given music piece.

We design feasible periodic motion patterns based on a model of the quadrocopter, which describes the dynamic capabilities of the vehicle. Control algorithms based on the vehicle model stabilize the vehicle in the air and guide it along the desired flight paths. However, without additional adaptation algorithms, the quadrocopter does not follow the desired path with the required accuracy resulting in a motion that is not in sync with the music. To perfect the vehicle's flight performance, measurements obtained from flight experiments are used to adapt the motion parameters sent to the vehicle ('commanded trajectory' in Fig. 4.2). This adaptation can be done online (during a flight performance) or offline (before a flight performance). The results are

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This chapter summarizes results that have previously been published in [1–5]. Parts of those papers are reproduced here for the sake of completeness..

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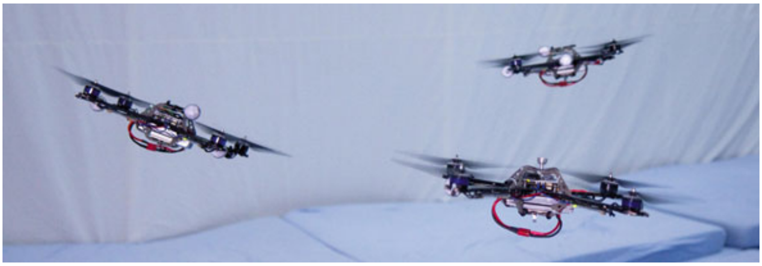

Fig. 4.1 A flight performance of multiple quadrocopters timed to music. (*Photo* Federico Augugliaro)

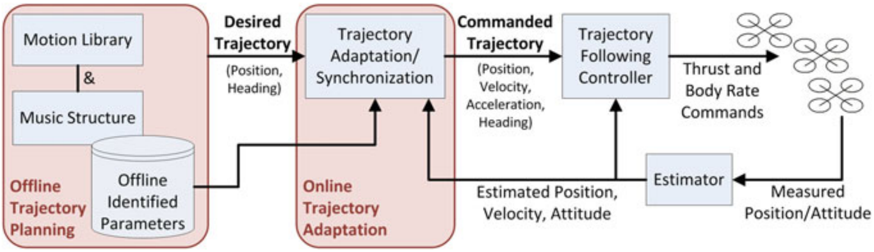

Fig. 4.2 High-level control architecture used for implementing rhythmic flight performances. Key components are the offline trajectory planning and online trajectory adaptation. (Position, velocity, and acceleration refer to the translational coordinates and heading corresponds to the vehicle yaw)

flight maneuvers that closely follow the desired periodic motion pattern ('desired trajectory' in Fig. 4.2) and align with the beat of the music.

This work can be viewed as a proof-of-concept result that shows the feasibility of rhythmic flight and represents an important step toward our vision of creating multivehicle aerial 'dance' performances.

4.1.1 Vision of a Quadrocopter Dance Performance

It takes an athlete to dance. But it takes an artist to be a dancer.
—Shanna LaFleur

Quadrocopters are exceptionally agile and "athletic" vehicles, but it takes more than agility to create a musical flight performance that is both viable and convincing. We envision a troupe of quadrocopters flying together across a big open stage—their movement choreographed to the rhythm of the music, their performance coordinated and skilled, and their choreography well-suited to their abilities and to the character of the music. A quadrocopter "dance".

A preliminary framework for designing and executing coordinated flight choreography to music has been implemented at the ETH Flying Machine Arena (Fig. 4.3).

Fig. 4.3 The ETH flying machine arena. *Left* Schematic drawing showing the motion capture camera system that provides accurate measurements of the vehicle's six degrees of freedom, position, and attitude. *Right* Photo of the installation at ETH Zurich. (*Photo* Raymond Oung)

In this framework, the underlying vehicle control is done automatically, while the high-level motion design is left to a human "choreographer". This work-in-progress currently enables the human operator to generate choreographies by assigning motion elements to individual music segments that correspond to the music's character. In addition to the algorithms presented herein, support is provided by, for example, a library of predefined, parameterized motion elements and a collision-free trajectory generator, which can be used for smoothly connecting single motion elements. Video sources of various quadrocopter flight performances are found at www.tiny.cc/MusicInMotionSite.

4.1.2 Artistic Motivation

> *As robots have grown more advanced, they have become our mirrors, as we watch the way they perform activities that we do as well. And as we watch, secrets are unlocked—secrets about how we, housed in our own biological frameworks, operate.*
>
> —Rodney Brooks, roboticist and entrepreneur

The embodied mind thesis [6], which straddles such diverse fields as philosophy, psychology, cognitive theory, neurobiology, robotics, and artificial intelligence argues that all aspects of cognition are shaped by the experiences of the body; that how we perceive the world around us (through our sensory system) and how we move through and interact with this world (through our motor system) intrinsically determines the ways in which we think and experience. Proponents of "embodied AI", such as Rodney Brooks [7] and Rolf Pfeifer [8], argue that for machines to be truly intelligent, they must have sensory and motor skills, and be connected to the world through a body.

It is interesting to consider the idea of "embodiment" also from the perspective of professional dancers, choreographers, and athletes—people for whom the ability to sense and move in the world forms a critical part of their work. In a paper entitled

"The Dance: Essence of Embodiment" [9] the philosopher/dancer duo Betty Block and Judith Lee Kissell describe dance as an "embodied way of being-in the-world," and that "an analysis of dance is a profoundly enriching way to better understand embodiment itself." In other words, to dance is to be an expert in embodiment.

It is no wonder, then, that robotics researchers have turned to dance as a means of understanding gesture and movement. Examples are provided in the subsequent Sect. 4.1.3.2. Note that many of these robotic/dance experiments involve humanoids and/or robotic arms that mimic human limbs. Indeed, mimicry is a proven means of generating understanding: much can be learned by reverse-engineering human movements and gestures.

But what happens when the "body" is not human? When the body is no longer constrained by the limits of arms, legs, torso, and head? In this research project, where quadrocopters learn and perform "dance", mere mimicry of human movement is no longer sufficient. A whole new meaning of "embodiment" begins to emerge.

It is obvious that the quadrotor body is mechanically different from the human body. It does not have arms, legs, or a head, but instead has rotating blades. Because it flies, it occupies three-dimensional space in a way that we humans cannot. Its movements are fundamentally different from ours: while we generate movement by pushing off a hard surface (such as the ground), a quadrocopter creates movement by "pushing" on air. These fundamental differences make it a challenge to design motions for quadrocopters that can be recognized as dance by humans, and that can been interpreted by human eyes as being "expressive".

Yet for all these differences, when it comes to dance performance, quadrocopters and humans share much in common as well. First and foremost, "dance"—whether performed by humans or by quadrocopters—is an exploration of three-dimensional space that must respect the boundaries of both the performance space and the body of the performer. Both humans and quadrocopters have limits to their abilities, and not every sequence of movements is feasible. In human dance, during a ballet barre exercise, for example, a Développé movement does not follow logically from a Plié (see Chap. 9); for quadrocopters, subsequent movements require smooth transitions without jumps in the vehicle position or attitude. Rhythmic ability is another feature shared by both humans and quadrocopters: when music is present, human motion is easily adapted to its meter, and with beat extraction software, this feat is accomplished by quadrocopters, too. Another commonality is the ability to dance in groups: human dance performances often feature troupes of dancers interacting with each other in a shared space; advances in trajectory planning allow quadrocopters to also share a space in a coordinated fashion without fear of collision. Humans also practice to perfect their skills—something we can enable in quadrocopters as well using parameter learning schemes [4]. And finally, humans teach and learn from each other; while cooperative machine learning remains to be explored in-depth, current research in this area is promising and suggests that shared learning could greatly enhance the learning process.

For robotics researchers, it is these commonalities that make an experiment in quadrotor dance so interesting. If the mechanical differences between humans and quadrocopters make it challenging for us to see them as "dance objects" or “dancers”, these differences are also what make quadrocopters capable of exploring and experiencing three-dimensional space in a way that humans physically cannot. For example, quadrocopters can engage with the three-dimensional space of the stage, including its full height, and can leverage air to generate movement—feats no human can do. In other words, what is challenging about quadrotor dance is also potentially liberating: when humans interface with quadrocopters by composing and executing quadrotor choreography, it opens up a new means of extending our own bodies into new physical and technological worlds.

Seen in this light, quadrocopters could become our dance partners, and the human–machine interface could become the cybernetic means through which we extend ourselves into new ranges of space and motion. This project is a first step toward that vision.

4.1.3 The Interplay of Dance and Technology

> *Dance and technology can shake hands but not at the expense of forgetting the essence of dance.*
> —Tero Saarinen, dancer and choreographer

The interplay of dance and technology has long been a space for experimentation, and a source for inspiration, innovation, and new developments. While technology has provided new means for dance expression and challenged dancers to rethink their art, dance has often challenged the state of the art of technology and motivated new technological developments. An early example is the theatrical lighting pioneered by the dancer Loie Fuller in the 1890s. Loie Fuller incorporated multicolored light in her performances and established stage lighting as a new dimension for dance expression. In addition, Fuller’s work pushed the boundaries of current technology and resulted in several patents related to stage lighting technology.

4.1.3.1 Information Technology and Dance

In the past 50 years, computer and information technology have influenced and transformed dance. The term “dance technology” has become a synonym for the relationship between dance and information technology [10, 11]. Attracted by the potential of this new field, dance performers, teachers, choreographers, and computer scientists have explored the partnering of two disciplines that are, as stated in [10], quite different: “Dance and technology make seemingly odd partners. Dance is the most ethereal of art forms and computer technology perhaps the most concrete of sciences. Whereas technologists deal with the logical, the scientifically verifiable, dancers, as artists, deal with the illogical, i.e. inspiration and finding truth in that which cannot be spoken.”

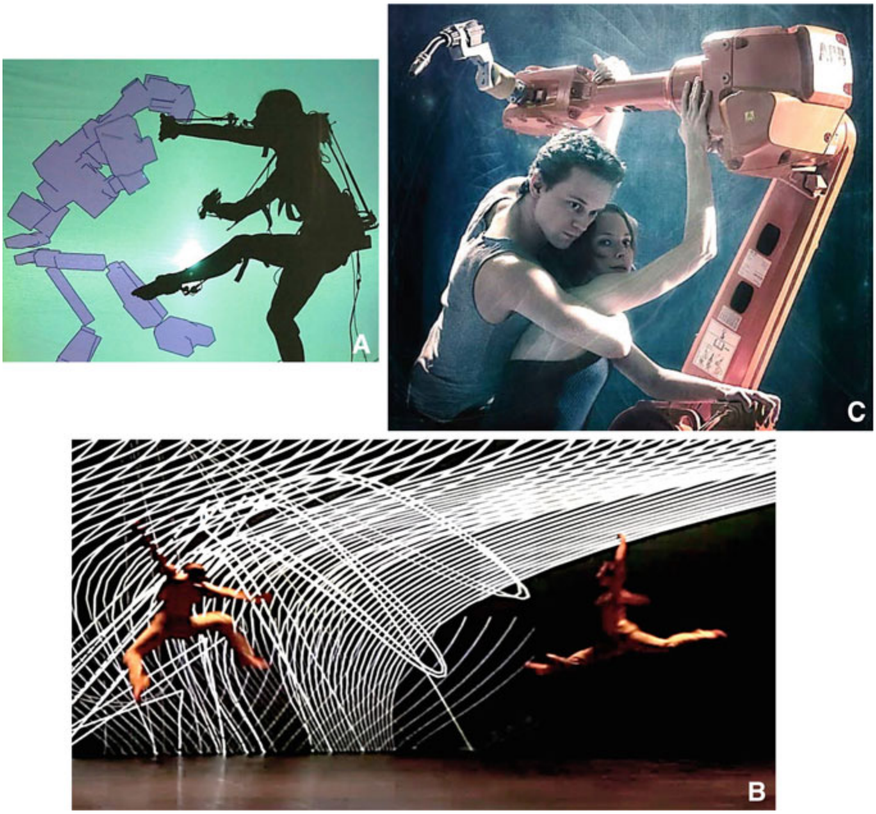

Fig. 4.4 The interplay of dance and technology. **a** Performance *100d11A0N1C00E1* of Carol Cunningham, April 2003: real-time animated projection of dancers' movement onto three large screens using motion capture (*Photo* David Umberger); **b** Performance *Apparition* of Klaus Obermaier and Ars Electronica Futurelab (www.exile.at), Ars Electronica 2004: interaction of dance and multimedia with real-time visual content generation; **c** Performance *Human interface* of Thomas Freundlich, May 2012: two dancers and two industrial robots perform together (*Photo* Johanna Tirronen)

Work at the interface of information technology and dance has advanced both disciplines by (i) integrating dance, emotions, and human character into computer technology and animations, and (ii) establishing new analysis tools and means of expression for dance performances.

Work in (i) has focused on computer graphics applications and aimed to create human-like virtual characters that are able to dance. Human motion capture data has been used to understand, model, and *imitate* human dance behavior [12, 13].

In (ii), information technology has led to new methods for expressing, creating, assessing, and instructing dance (cf. [14, 15]). Results include interactive performances of human dancers with computer-generated sound and images (e.g., computer-animated virtual dancers projected on a wall) [15–17], responsive environments where the dancer's movement controls video, audio, and/or lightning [18, 19], computer-assisted choreography design based on a language for human movement representation and on virtual dance animation [11, 20], motion tracking and capture to record or teach a piece [21], and multimedia in dance education [22, 23]. Figure 4.4, photos a and b show two different stage performances that explored the technological possibilities for new forms of dance expression. Obermaier says [24], "The goal was to create an interactive system that is much more than simply an extension of the performer, but is a potential performing *partner*". Carol Cunningham summarizes her work as follows, "Motion capture is another tool for expression. The image may be on screen and generated by technology, but it's an *extension of the body*". Resembling human movement (Fig. 4.4a) and extending human motion into new spaces (Fig. 4.4b) were goals of the dance and technology partnering with the following outcome [18]: "The new convergences between dance and technology reflect back on the nature of dance, its physical-sensory relationship to space and the world, its immediate, phenomenological embodiedness, its lived experience in one place".

4.1.3.2 Robotics and Dance

As technology has advanced in the last 10 years and robots have become more approachable, they have found their way into dance just as information technology has done before. The physical embodiment of robots and their abilities to interact provide a new means for dance expression as well as for studying human–robot interaction and human dance.

Research on robotics and dance has come a long way: from building robots that are capable of executing human-like motions and enabling them to imitate human dance, to enabling robot–human interactions in dance performances, adapting robot dance to human preferences, and understanding human dance through robots. As dance has previously been a human-centered discipline typically designed, performed, and evaluated by humans, research into "dancing robots" has primarily dealt with humanoid robots and aimed for human-like behavior in robots. In this work, we consider a new embodiment—a group of flying robots—but still face similar questions such as: What is dance? What do humans recognize as dance? Which algorithms enable dance-like behavior in robots?

First approaches toward robotic dance of humanoid robots tried to imitate human dance motion. In [10] basic dance motions for a robotic arm were designed using choreographic elements from human dance such as shape, space, time, and force. For humanoid robots, data from human demonstrations (obtained using a motion capture system) was used to define basic robot dance motions, which—when concatenated—create full robot choreographies [25–28]. A perfect example for human imitation is a female android created by Japanese roboticists, which sings and dances along with a troupe of humans.[1] The android's motion was created by a professional choreographer using a tool proposed in [29].

Recent work aims to understand the rules of human dance, which may ultimately lead to a larger robot autonomy when executing dance. One approach is based on the concept of human dance styles and detailed in the Chap. 9 of this book. Instead of robots that follow preprogrammed motions, various styles of human movement are defined, which in turn can be reproduced on a humanoid robot by generating sequences of motions autonomously based on underlying rules. Other approaches presented in this book try to understand human flocking in order to derive multiagent behavior (see Chap. 2) and the human communication through movements (Chap. 3). Other concepts that could explain what humans recognize as dance are skill-based approaches [30] (defining fundamental joint relationships such as Opposite, Symmetry and Formation and learning likable sequences from human feedback), effects related to motion synchrony and timed repertoire changes [31], and automatic motion selection based on musical mood [32] or musical emotions [33, 34].

Moreover, researchers currently investigate the interaction between humans and their robotic counterparts, and potential adaptation schemes for robots. The adaptation of a robot's dance behavior to human preferences is described in [35]. In [36–39] the rhythmic behavior of the robot adapts to the human based on appropriate estimation techniques that predict the human motion. Stage performances focusing on the human–robot interaction include Thomas Freundlich's performance in Fig. 4.4c and also the work in Chap. 9 of this book. Moreover, recently two artistic performances have featured quadrocopters on stage with human actors/dancers [40, 41]; these focused on the interplay between humans and machines, and had skilled human operators for controlling the quadrocopters.

4.1.3.3 Relationship to Our Work

The history of technology and dance provides a great context for our experiment, where the performers of the dance are a swarm of quadrotor vehicles. Their flight capabilities may offer—similarly to how humanoid robots have done before—new means of dance expression, including motions in the full three-dimensional space. New challenges result from the nonhuman-like body shape and motion characteristics. While work on humanoid robots has largely imitated human dance behavior, choreographies for quadrocopters must rethink the question, "What do humans recognize as dance?," and define quadrotor motions accordingly. Nevertheless, ideas for human dance choreography (such as shape, space, time and force) and concepts developed for humanoid robots may partially apply and/or may be a great source for inspiration. Overall, by studying concepts and algorithms for creating "dance-like" performances (including human–robot interaction, adaptation to the human behavior or motion planning) not in the context of the human body may enable us to understand more generally what makes robots move in a way that humans can relate to.

[1] Video found at http://youtu.be/3JOzuTUCq6s.

4.1.3.4 A Final Note

Human dance has proven to be an inspiration for technology developments. Moreover, technology has proven to extend the vocabulary of dance creation and performance to an extent that we are often not aware of. An example of the tangible connection between robots and humans is a (human) dance style called "robot dance" that became popular in the 1980s and that attempts to imitate a dancing robot. The style is characterized by jerky mechanical movements. Inspiration inevitably goes both ways: from human dance to technology and from technology to human dance. Just as our project is a robotics research experiment, it is also an experiment in dance and choreography.

4.1.4 First Steps Toward a Rhythmic Flight Performance

Art challenges technology, and technology inspires art.
—John Lasseter, chief creative officer at Pixar and Walt Disney Animation Studios

John Lasseter's quote reflects the character of many past contributions at the interface of dance and technology (cf. Sect. 4.1.3). It also provides the context for our work toward a rhythmic flight performance of multiple quadrocopters. While the technological capabilities available today (such as small-sized off-the-shelf flying robots) inspired us to think about "dancing quadrocopters" in the first place, implementing an aerial choreography challenged the current knowledge in multivehicle autonomous flight and led to novel research results, cf. [1–4, 42].

In this chapter, we focus on the research questions that are at the core of the proposed project. We show how control theory can be used to approach these questions analytically, and offer an intuitive explanation of our findings.

In particular, the topics investigated in this book chapter are:

1. Quadrocopter Dynamics: How do quadrocopters move? Which motions are possible with quadrocopters?
2. Motion Design: How to generate "dance-like" quadrocopter motions?
3. Motion Feasibility: Which motions are feasible given the actuator and sensor constraints of the vehicle?
4. Quadrocopter Control: How do quadrocopters execute their movements?
5. Motion Synchronization: Can quadrocopters move in the rhythm of the music? How well can they perform a rhythmic motion?
6. Rhythmic Performances: What has been accomplished to date?

The above questions are driven by the goal of creating a rhythmic flight performance. The answers to these questions are obtained from control theoretic analysis and design.

It is also interesting to make the connection to Chap. 5 here, where similar questions are considered for a different system, namely robotic marionettes, and tools from controls are used to address the issue of feasibility, motion planning, and timing. An opposite approach is taken in Chap. 7, where music is generated from motion, where synchronization of motion and music plays an equally important role.

4.2 Quadrocopter Dynamics: How do Quadrocopters Move?

Dance is the language of movement. It is the realization of the body's potential as an instrument of expression.
—Victorian Board of Studies Dance study design, 1994

Human dance expression is fundamentally tied to the human body and its physical capabilities. As an "instrument of expression", the human body seems to enable an endless range of different movements and different movement qualities. Just imagine how many poses there are for a human (without even considering movement): we can stand with two feet on the ground and various hand, arm, finger, and head positions, and can make an almost infinite number of facial expressions. Moreover, skilled dancers can stand still on just one leg... The number of degrees of freedom of a human body (that is, the number of independent joints and possible directions of rotation in those joints) is large but nevertheless motions are constrained by the limits of arms, legs, torso, and head.

In comparison, for a quadrocopter (see Fig. 4.5) there is only one position that allows it to stand still; namely, being horizontal in the air and producing an upward force with its propellers that is equivalent to the gravitational force acting on the vehicle. Moreover, a quadrocopter has only six degrees of freedom: three translational (its three-dimensional position) and three rotational (its attitude), see Fig. 4.5b. However, with only four independent motors (Fig. 4.5a), quadrocopters are underactuated; that is, rotational and translational motion cannot be controlled independently but are coupled [43]. More insight into the coupling will be provided below, where we derive a model for the quadrocopter dynamics from first principles and also specify the constraints of the vehicle. The dynamics model and constraints define the dynamic capabilities of the vehicle in mathematical terms. We provide an interpretation of the findings with respect to our goal of generating rhythmic flight performances.

4.2.1 Dynamics Model of the Quadrocopter

The quadrocopter is described by six degrees of freedom: the translational position $s = (x, y, and z)$ measured in the inertial coordinate system **O** and the rotational position (also called 'attitude') represented by the rotation matrix $\mathbf{R}(t)$ from the body frame **V** to the inertial frame **O** as shown in Fig. 4.5b.

The translational acceleration of the vehicle is dictated by the attitude of the vehicle and the total thrust produced by the four propellers. The translational motion

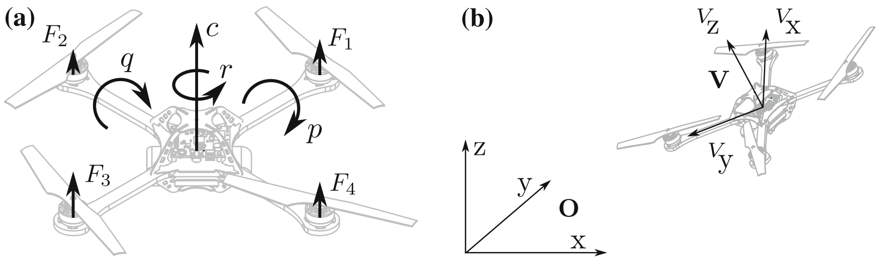

Fig. 4.5 **a** Control inputs of the quadrocopter. **b** Quadrotor position and attitude. Schematic of the quadrocopter with: (**a**) the control signals sent to the vehicle being the body rates p, q, and r and the collective thrust c, and (**b**) the quadrotor position and attitude **V** defined with respect to the inertial coordinate system **O**. These control signals are converted by an onboard controller into motor forces F_i, $i \in \{1, 2, 3, 4\}$

of a quadrocopter in the inertial frame **O** is described by

$$\begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{z}(t) \end{bmatrix} = \mathbf{R}(t) \begin{bmatrix} 0 \\ 0 \\ c(t) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \Leftrightarrow \begin{array}{l} \ddot{x} = c\, b^x \\ \ddot{y} = c\, b^y \\ \ddot{z} = c\, b^z - g \end{array}, \tag{4.1}$$

where g is the acceleration due to gravity and $c(t)$ is the collective thrust; that is, the sum of the rotor forces F_i normalized by the vehicle mass m,

$$c = \tfrac{1}{m} \textstyle\sum_{i=1}^{4} F_i. \tag{4.2}$$

The motor forces F_i, $i \in \{1, 2, 3, 4\}$, represent the inputs to the quadrocopter (see Fig. 4.5). The values (b^x, b^y, b^z) correspond to the third column of the rotation matrix, namely $(\mathbf{R}_{13}, \mathbf{R}_{23}, \mathbf{R}_{33})$, and represent the direction of the collective thrust in the inertial frame **O**.

Each rotor produces not only a force F_i, $i \in \mathcal{I} = \{1, 2, 3, 4\}$, in the positive ${}^{V}z$ direction, but also a reaction torque M_i perpendicular to the plane of rotation of the blade, see Fig. 4.5a, where

$$M_i = kF_i, \quad k = \text{const}, \tag{4.3}$$

describes the relationship between the motor force F_i and the associated reaction torque M_i. The parameter k is given by the motor characteristics, see [43] for details. Rotors 1 and 3 rotate in the negative ${}^{V}z$ direction, producing a moment that acts in the positive ${}^{V}z$ direction; while rotors 2 and 4 rotate in the opposite direction resulting in reaction torques in the negative ${}^{V}z$ direction. Given the inertia matrix I with respect to the center of mass and the vehicle frame **V**, the rotational dynamics of the body-fixed frame are given by

Table 4.1 Quadrocopter parameters

	Definition	Value
m	Mass of vehicle	$0.468\,\mathrm{kg}$
L	Vehicle arm length	$0.17\,\mathrm{m}$
I_x	Inertia around vehicle ${}^{V}x$-axis	$0.0023\,\mathrm{kg\,m^2}$
I_y	Inertia around vehicle ${}^{V}y$-axis	$0.0023\,\mathrm{kg\,m^2}$
I_z	Inertia around vehicle ${}^{V}z$-axis	$0.0046\,\mathrm{kg\,m^2}$
k	Motor constant	$0.016\,\mathrm{m}$
$F_{\min}$	Minimum rotor force	$0.08\,\mathrm{kg\,m/s^2}$
$F_{\max}$	Maximum rotor force	$2.8\,\mathrm{kg\,m/s^2}$

$$I\dot{\Omega} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ k(F_1 - F_2 + F_3 - F_4) \end{bmatrix} - \Omega \times I\Omega, \tag{4.4}$$

where $\Omega = (p, q, r)$ represent the quadrocopter angular body velocities around the body $({}^{V}x, {}^{V}y, {}^{V}z)$ axes and L is the distance from each motor to the center of the quadrocopter. The vehicle's principal axes coincide with the vehicle frame axes resulting in a diagonal inertia matrix with entries (I_x, I_y, I_z), where $I_x = I_y$ because of symmetry.

The rotation matrix $\mathbf{R}$ evolves according to (cf. [44])

$$\dot{\mathbf{R}}(t) = \mathbf{R}(t) \begin{bmatrix} 0 & -r(t) & q(t) \\ r(t) & 0 & -p(t) \\ -q(t) & p(t) & 0 \end{bmatrix}, \tag{4.5}$$

In our setup, an onboard controller closes the loop on the angular body velocities Ω using onboard gyroscope measurements. As a result, the control signals sent to the the quadrocopter are the collective thrust command c_c and the commanded angular body velocities $\Omega_c = (p_c, q_c, r_c)$, see Figs. 4.2 and 4.9. Based on the commanded values (Ω_c, c_c) and the gyroscope measurements, the onboard controller calculates the required motor forces $F_i,\ i \in \mathcal{I}$.

The specific vehicle parameters for the quadrocopters used in this work (see Fig. 4.1) are given in Table 4.1.

4.2.2 Vehicle Constraints

The agility of the quadrocopter is constrained by the minimum and maximum force of a single motor, $F_{\min} \le F_i \le F_{\max},\ i \in \{1, 2, 3, 4\}$, with $F_{\min} > 0$, since the motors cannot reverse their direction. The collective thrust is bounded by

$$c_{\min} \le c \le c_{\max} \quad \text{with} \quad c_{\min} = 4\,F_{\min}/m,\ c_{\max} = 4\,F_{\max}/m. \tag{4.6}$$

In addition, due to the motor dynamics the rate of change of the motor forces is bounded in reality and the turn rates must be bounded because of the limited measurement range of the gyroscopes used for onboard vehicle control. We neglect both limitations in the following sections to simplify the presentation. The bounds of the thrust rate $\dot{F}_i$ and the turn rates Ω are high (23.9 kg m/s^3 and 25 rad/s, respectively) and do not significantly affect the results in Sect. 4.4.

4.2.3 *Implications for a Rhythmic Flight Performance*

The above equations describe the motion capabilities of a quadrotor vehicle. From (4.1) we see that the vehicle acceleration is always perpendicular to the plane of the rotors; that is, for a motion in the x, y-direction the quadrocopter must tilt. The translational and rotational degrees of freedom are, therefore, coupled and cannot be specified independently. A rotation of the quadrocopter is achieved by sending appropriate turn rates Ω_c, see (4.5). The rotational dynamics around the ${}^V x$- and ${}^V y$-axes are symmetric, see (4.4), and fast due to the low rotational inertia terms (Table 4.1).

One set of independent motion parameters for a quadrocopter is its *three-dimensional position over time* and the *evolution of the heading angle*, cf. [43] and Fig. 4.7. Compared to the human body, the "body's potential" of a quadrocopter for expressive movements is therefore limited to the position and heading in space over time. Finding motion patterns that are convincingly expressive to the human eye is not trivial and is discussed in Sect. 4.3.

4.3 Motion Design: What is a Dance Step for a Quadrocopter?

Dance is a poem of which each movement is a word —Mata Hari, dancer

As human dance choreography is typically described by sequences of basic movements, we expect a flight performance of quadrocopters to be composed of basic motion elements that—when combined into sequences—allow for a multifaceted, meaningful quadrocopter choreography. As a first step, our goal is to develop basic, rhythmic motion elements that can be executed by quadrocopters and timed to the music beat. These basic rhythmic flight motions represent the "words" that may later tell a "poem".

Periodic motions are a natural human response to hearing a recurring music beat: we often clap, sway, or tap our feet when we hear music. In our research, we want the flying vehicle to mimic this behavior. Periodic motion elements thus represent the basic building blocks of our choreography. As highlighted above, the degrees of freedom of a quadrocopter motion are restricted to the three-dimensional position and its heading. We therefore develop motion elements that show a periodicity in the

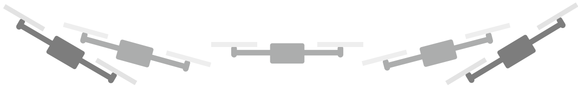

Fig. 4.6 A periodic side-to-side motion with the music beats occurring at the outermost points of the movement

vehicle position and use the vehicle's agility to achieve temporal variety. Below we present a parameterized motion description that enables motion variations that are indispensable when aiming for an expressive choreography.

4.3.1 Music Analysis

The more you understand the music, the easier you can dance.
—Orlando Gutinez

Fundamental to our goal of creating rhythmic flight movements is a tight connection of motion and music. We therefore analyze the music first and then assign appropriate motions to the vehicle. The goal of the music analysis is to extract music features and their respective time signatures. The result is a vocabulary that describes the song's temporal development. We use this time information to assign suitable quadrocopter motions to different sections of the song.

In order to achieve rhythmic behavior, we are particularly interested in the music beat, which represents the basic rhythmic unit of a song and plays a prominent role in the motion design. Currently we use the *BeatRoot* software tool [45] to extract beat times from a song. We store music beats and their respective start times in a text file. This information is then used to create matching flight trajectories; for example, movements that reflect the music tempo.

A simple example that highlights the key idea is shown in Fig. 4.6: the quadrocopter performs a planar side-to-side motion where, at beat times, the vehicle reaches the outermost points of the motion, either on the left or right.

4.3.2 Periodic Motions

When you dance, your purpose is not to get to a certain place on the floor. It's to enjoy each step along the way. —Wayne Dyer. author

We specify basic, rhythmic motion elements as the *evolution* of the quadrocopter's translational position in three dimensions $s_d(t) = (x_d(t),\ y_d(t),\ z_d(t))$ and its heading $\psi_d(t)$ over time. We introduce parameterized motion primitives

$$s_d(p,t), \quad \psi_d(p,t), \tag{4.7}$$

which depend on a set of adjustable motion parameters p and are defined over a finite time interval $t \in [t_0, t_f] \subset \mathbb{R},\ t_f < \infty$. Parameterized motion primitives allow for variety and expressiveness in the choreography design. Consider a horizontal circle, for example, where the radius, speed of rotation, and center point can be adapted depending on the use case. Note that the vehicle heading ψ_d can be designed independently of the position and is not explicitly considered in the following.

Our objective is to offer a similar range of motions as is used in human dance composition. In this context, we ask: Which choices does a professional dance choreographer have when creating a performance? How can we provide the tools and degrees of freedom necessary for implementing an expressive performance on the quadrocopter?

Four fundamental choreographic elements—time, space, energy, and structure—are commonly used by professional dancers, choreographers, and dance teachers to build choreography with interest, dynamics, and estethic appeal, cf. [46, 47]. These parameters provide a framework for meaningful quadrocopter choreography, and are described as follows:

Space Space refers to the area the dancer is performing in. It also relates to how the dancer moves through the area, as characterized by the direction and path of a movement, as well as its size, level, and shape.

Time Time encompasses rhythm, tempo, duration, and phrasing of movements. Using time in different combinations can create intricate visual effects. Ideas such as quick-quick, slow, or stop movements are examples.

Energy Energy relates to the quality of movement. This concept is recognizable when comparing ballet and tap dance. Some types of choreography are soft and smooth, while others are sharp and energetic.

Structure Structure represents the organization of movement sequences into larger concepts: the combination and variation of movements using recurring elements, contrast, and repetition. Movements can even follow a specific story line to convey certain information through a dance.

Examples illustrating the four elements of dance are found in [46, 47].

One way of introducing parameterized, rhythmic motion primitives that capture a wide range of different movements is as a Fourier series [48],

$$s_d(t) = a_0 + \sum_{k=1}^{N} a_k \cos(k\,\omega_d t) + b_k \sin(k\,\omega_d t)\,, \tag{4.8}$$

where $\omega_d = 2\pi/T$ represents the fundamental frequency corresponding to a music beat of frequency $1/T$, where beats are T seconds apart. Additional design parameters are the constant vectors $a_0, a_k, b_k \in \mathbb{R}^3,\ k \in \mathcal{K} = \{1, 2, \ldots, N\}$, and $N \geq 1$; that is, $p = \{\omega_d, N, a_0, a_k, b_k \mid k \in \mathcal{K}\}$. The parameters characterize the desired translational position $s_d(t)$ of the quadrocopter and allow us to express the key choreographic elements:

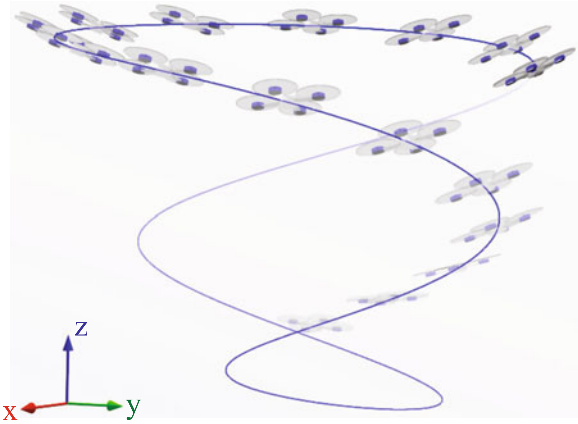

Fig. 4.7 An example of a periodic motion primitive studied in this chapter

Space The parameters a_0 and $a_k, b_k,\ k \in \mathcal{K}$ define the amplitudes of the periodic motion and, thus, the spacial dimension of the movement. These vectors also specify the direction of the motion and the overall three-dimensional shape of the curve.

Time The underlying rhythm is given by the frequency ω_d. When the choreography is set to music, the frequency ω_d is related to the music's tempo. Different tempos can be combined when choosing $N > 1$. The overall duration of the motion can be adjusted via t_f.

Energy The higher the value of N, the more energetic and sharp are the possible motions, cf. [48].

Structure The motion primitives described in (4.8) can be combined into sequences, which can in turn be combined to create an overall choreographic performance. Endless permutations are possible, much the way individual words can be combined into a variety of sophisticated stories, or a series of gestures can be combined to reveal a performer's mood or emotion to an audience.

In short, the general motion description (4.8) reflects the fundamental choreographic elements and allows for a multidimensional choreography. Out of the variety of motions captured by (4.8), Fig. 4.7 illustrates the one with $N = 3$, $T = 10$, $a_0 = (0, 0, 3)$, $a_1 = (0, 0, 1)$, $a_2 = (1, 0, 0)$, and $b_3 = (0, 1, 0)$, and a_3, b_1, b_2 being zero. A Matlab file for generating arbitrary motion primitives of the proposed type is available online at www.idsc.ethz.ch/Downloads/QuadDance.

In order to make (4.7) and (4.8) a useful tool for choreographers, we need to specify which motion primitives can be realized on the vehicle. The dynamics and physical limits of the quadrocopter define the feasible sets of parameters p. This is done in the next section.

4.4 Motion Feasibility: What are the Physical Limits of a Quadrocopter?

The dancer is restricted by self-limits, the limits of being in this body with these abilities and not others.

—Sondra Horton Fraleigh in "Dance and the Lived Body: A Descriptive Aesthetics" [49]

Though dancers and athletes are trained to push the physical limits of their bodies to extremes, they nonetheless remain constrained by the rules of physics. Quadrocopters, too, are limited by their body's dynamics (Sect. 4.2). For example, our quadrocopters cannot keep a constant height when flying sideways with angles larger than 66° (cf. Fig. 4.6). To create a "choreography" for quadrocopters, we must be aware of and account for these physical limitations.

Below we describe a method for checking the feasibility of quadrocopter motions. The approach, meant as a validation tool for preprogrammed quadrocopter performances, is based on the first principles models in Sect. 4.2 and ensures that a desired trajectory respects both vehicle dynamics and motor thrust limits. The goal is to determine sets of motion parameters p, cf. (4.7), (4.8), that represent rhythmic motions that can be realized with a quadrocopter. The result of this analysis is a library of feasible motion primitives that can be used to create multifaceted performances.

4.4.1 Motor Thrust Limits

For the subsequent feasibility analysis, we assume that motion primitives, cf. (4.7), are twice-differentiable in time. This assumption is satisfied for the periodic motions primitives (4.8) introduced in the previous section. Feasibility is formulated in terms of the collective thrust limits $(c_{\min}, c_{\max})$ and the motion parameters p. The objective is to derive a set of inequalities that specify feasible parameter sets p given the limits $(c_{\min}, c_{\max})$.

For a desired motion primitive s_d, we rewrite (4.1),

$$\mathbf{R}\, n\, c_d = \ddot{s}_d + n\, g, \tag{4.9}$$

where $n = (0, 0, 1)$ and c_d is the nominal thrust input required to achieve s_d. Taking the 2-norm, we can solve for c_d, $c_d \geq 0$,

$$\|\mathbf{R}\, n\, c_d\| = \|\ddot{s}_d + n\, g\| \quad \Leftrightarrow \quad c_d = \|\ddot{s}_d + n\, g\| \ . \tag{4.10}$$

Recalling that $s_d = s_d(p, t)$ and (4.6), the inequalities guaranteeing the maximum and minimum bounds of the collective thrust are

$$c_{\min} \leq \|\ddot{s}_d(p, t) + n\, g\| \leq c_{\max}, \quad t \in \left[t_0, t_f\right]. \tag{4.11}$$

This feasibility requirement can be checked for any given desired motion primitive $s_d(p,t)$ by calculating its second time derivative. No further calculations are necessary. In particular, the nominal quadrocopter inputs associated with $s_d(p,t)$ need not be determined in advance. The inequalities (4.11) exclude the majority of infeasible parameters p and help to build an intuition as to what is feasible for a quadrotor vehicle.

In order to be more precise, single motor constraints and turn rate constraints must be considered, cf. Sect. 4.2.2. For those constraints explicit parameter-dependent inequalities are generally difficult to derive (see [3] for details). Instead, in our current software framework, we numerically assess the feasibility of a created motion sequence before actual flight, see [50].

4.4.2 Example: Side-to-Side Motion

To demonstrate the above feasibility test, we consider a simple periodic motion that falls into the framework introduced in (4.8): a horizontal side-to-side motion as illustrated in Fig. 4.6. In fact, the side-to-side motion was the first rhythmic motion that we implemented on a quadrocopter and executed to music [1].

The planar side-to-side movement is given by

$$x_d(t) = A\cos(\omega_d t), \quad y_d(t) = z_d(t) = \psi_d = 0. \tag{4.12}$$

The side-to-side motion is a special case of the general motion primitive description (4.8), where $N = 1$, $a_1 = (A, 0, 0)$ and $a_0, b_1 = (0, 0, 0)$.

To determine feasible combinations of amplitudes A and frequencies ω_d, we calculate the second derivative of (4.12) and insert it into (4.11):

$$c_{\min} \leq \sqrt{A^2\omega_d^4\cos^2\omega_d t + g^2} \leq c_{\max}. \tag{4.13}$$

For a given pair (A, ω_d), these inequalities must be satisfied for all $t \in [0, T]$. Therefore, it is enough to consider the maximum and minimum values over T. We obtain

$$A\omega_d^2 \leq \sqrt{c_{\max}^2 - g^2} \quad \text{and} \quad c_{\min} \leq g. \tag{4.14}$$

The second inequality must be satisfied in order for a quadrocopter to be able to land. In brief, all parameter pairs (A, ω_d) satisfying the inequality (4.14) represent side-to-side motions that stay within the collective thrust limits (4.6).

For the vehicle parameters in Table 4.1, Fig. 4.8 illustrates the feasible set of side-to-side trajectories (A, ω_d). The dark gray region contains parameter sets that are infeasible due to the collective thrust limit, cf. (4.14). We also depict (light gray area) the parameter sets that become infeasible when taking into account the minimum and maximum force limits of each single motor (see Sect. 4.2.2); the corresponding

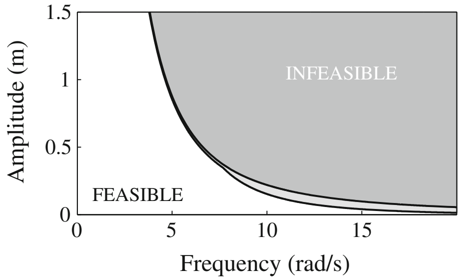

Fig. 4.8 Feasible parameter sets for the side-to-side motion primitive. The *dark gray* region denotes parameter sets that are infeasible due to collective thrust limits; *light gray* denotes additional parameter sets that are infeasible due to the minimum and maximum force limits of each single motor

derivations are presented in [3]. From Fig. 4.8 we see that if we want to perform two side-to-side motions per second ($\omega_d \approx 12.6$ rad/s), a motion amplitude of 0.5 m is clearly infeasible. We also see that for the side-to-side motion the single motor force limits exclude only a small additional number of parameter sets. The inequalities (4.11) represent a simple means to understand which motions are feasible.

4.5 Quadrocopter Control: How do Quadrocopters Execute Their Movements?

Technique—bodily control—must be mastered only because the body must not stand in the way of the soul's expression.
—La Meri, dancer and choreographer

In Sect. 4.3 we introduced rhythmic motion elements with the goal of enabling expressive choreography, where the movements were defined by the desired evolution of the quadrocopter position over time. However, similarly to human dancers who constantly work on perfecting their body control, quadrocopters require sophisticated control algorithms to guide their "bodies" along the desired trajectories. Just recall that the smallest mistake may lead to the vehicle falling out of the sky. In this section, we derive a motion controller that maintains the quadrocopter on the specified trajectory during actual flight.

4.5.1 Trajectory-Following Controller

The trajectory-following controller (TFC) accepts as input commanded positions, velocities, and accelerations, as well as, a yaw angle trajectory (cf. Fig. 4.2):

$$(s_c(t), \dot{s}_c(t), \ddot{s}_c(t), \psi_c(t)) \,. \tag{4.15}$$

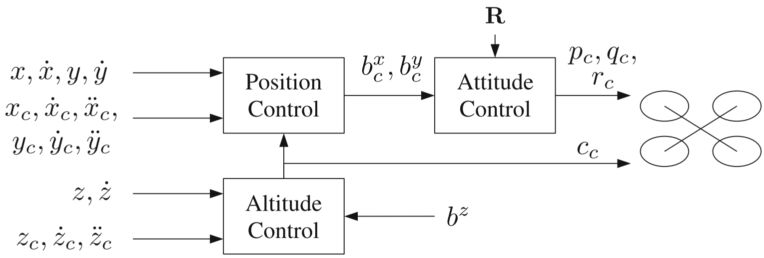

Fig. 4.9 Cascaded control loops of the trajectory-following controller (TFC)

We usually obtain appropriate input commands directly from the desired sequence of motion primitives (see Sects. 4.3 and (4.7)) by setting: $s_c(t) := s_d(t)$ and $\psi_c(t) := \psi_d(t)$. The respective time derivatives $\dot{s}_c(t)$, $\ddot{s}_c(t)$ are also computed from the desired trajectory $s_d(t)$, which is preplanned and known in its full length prior to flight.

The TFC is a standard component of our experimental testbed. Control is based on precise measurements of the vehicle position and attitude (in our case, provided by a motion capture system). The TFC receives the quadrocopter's position $s = (x, y, z)$, velocity $\dot{s}$ and attitude $\mathbf{R}$ from an estimator and, in turn outputs the body rate and collective thrust commands (Ω_c, c_c) to the vehicle, see Fig. 4.9. The TFC consists of three separate loops for altitude, horizontal position, and attitude. While the TFC operates in discrete time, the controller design is based on the continuous-time system dynamics representation.

The altitude control is designed such that it responds to altitude errors $(z - z_c)$ like a second-order system with time constant τ_z and damping ratio ζ_z,

$$\ddot{z} = -\frac{2\zeta_z}{\tau_z}(\dot{z} - \dot{z}_c) - \frac{1}{\tau_z^2}(z - z_c) + \ddot{z}_c. \tag{4.16}$$

It uses the collective thrust to achieve this. With (4.1) and (4.16), we obtain

$$c_c = (\ddot{z} + g)/b^z. \tag{4.17}$$

Similarly, the two horizontal position control loops are shaped based on (4.1) with c_c from (4.17). Commanded rotation matrix entries b_c^x, b_c^y result. The attitude control is shaped such that the two rotation matrix entries b^x, b^y react in the manner of a first-order system; that is, for x: $\dot{b}_c^x = (b^x - b_c^x)/\tau_{RP}$. This is directly mapped to the commanded angular body velocities (p_c, q_c) using (4.5) and the estimated attitude $\mathbf{R}$,

$$\begin{bmatrix} p_c \\ q_c \end{bmatrix} = \frac{1}{\mathbf{R}_{33}} \begin{bmatrix} \mathbf{R}_{21} & -\mathbf{R}_{11} \\ \mathbf{R}_{22} & -\mathbf{R}_{12} \end{bmatrix} \begin{bmatrix} \dot{b}_c^x \\ \dot{b}_c^y \end{bmatrix}. \tag{4.18}$$

Vehicle yaw control can be considered separately, since rotations around the body ${}^V z$-axis do not affect the above dynamics. The yaw controller is a proportional controller and the resulting yaw angle rate is mapped to r_c using the kinematic relations of

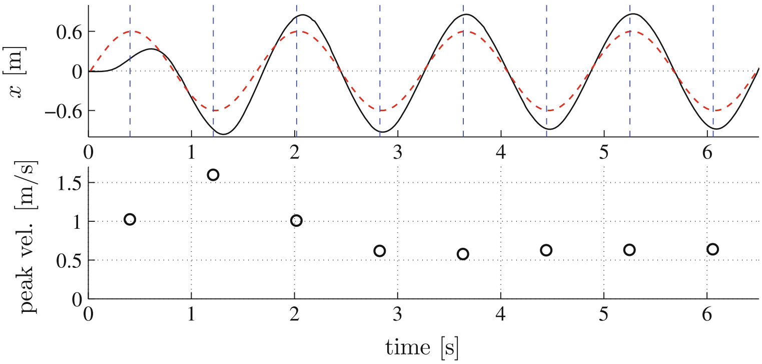

Fig. 4.10 Side-to-side motion, *no motion parameter adaptation*. *Top* quadrocopter response (*solid*) for a desired oscillation in the x-direction (*dashed*). *Bottom* corresponding peak velocities, i.e., absolute value of vehicle velocity at the peaks of the desired trajectory. High peak velocities imply a large phase error

Euler angles. The innermost loop, on board the quadrocopter, controls the angle rates (p, q, r) to the calculated set points (p_c, q_c, r_c).

In the ideal case, where the quadrocopter dynamics correspond to the model (4.1) and some other mild assumptions are made (see [4] for details), the derived controller yields perfect trajectory tracking. In summary, we have presented a control framework that enables an autonomous quadrocopter flight along a desired trajectory defining the vehicle position and the heading of the vehicle over time.

4.5.2 Tracking Performance of Periodic Motions

When using the derived TFC to track the side-to-side motion (4.12), we considered before with $s_c(t) := s_d(t)$ and $\psi_c(t) := \psi_d(t)$ (that is, the desired periodic trajectory is directly sent to the vehicle controller), we observe, at steady state, a sinusoidal motion of the same frequency with a constant error in amplitude and phase, resulting in asynchrony and spatial inaccuracies, as shown in Fig. 4.10 (top figure). The amplitude error of the quadrocopter response (black solid line) is obvious; the phase error between the reference trajectory and the actual quadrocopter response is hardly noticeable. However, small phase errors are very visible and audible in actual experiments as humans are especially sensitive to nonzero vehicle velocity at beat times (see [5] for more details). Correspondingly, the bottom plot of Fig. 4.10 illustrates the velocity of the quadrocopter at beat times; that is, when the reference trajectory reaches its maximum or minimum value.

For periodic motions in three dimensions, a similar behavior is observed: phase shift and amplitude error are observed in each translational direction and are not necessarily equal in size. In this case, the shape of the resulting motion can change.

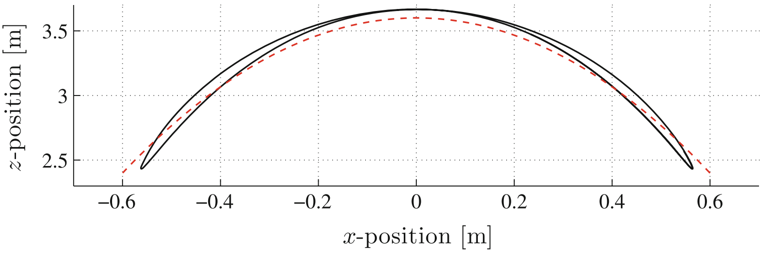

Fig. 4.11 Vertical bounce motion, *no motion parameter adaptation.* The vehicle's response (*solid*) can differ in shape from the desired trajectory (*dashed*)

For example, a desired bounce motion (Fig. 4.11) results in a bent eight-shaped vehicle motion.

In order to achieve precise temporal and spatial tracking, we adapt the parameters of the commanded trajectory (4.15) sent to the TFC in the next section. Later we see that these parameters can be identified/learned prior to the flight performance in order to effectively reduce initial transients.

4.6 Motion Synchronization: Can a Quadrocopter Move in the Rhythm of the Music?

I like to see you move with the rhythm; I like to see when you're dancing from within.
—Bob Marley, singer and composer

"Moving with the rhythm" is the ultimate goal of this work, where we aim to control the motion of quadrocopters to an external music signal. As highlighted in the previous section, pure feedback controlled to insufficient quadrocopter tracking with a noticeable phase and amplitude error.

The goal of this section is to prove the feasibility of a precise synchronization between quadrocopter motion and music, where we use the term "synchronization" loosely, inasmuch as it encompasses both spatial and temporal tracking accuracy. Our strategy for coping with the aforementioned constant phase shift and amplitude error is to adjust the motion parameters of the trajectory commanded to the underlying trajectory-following controller (see Fig. 4.2). This means, for example, that if the amplitude of the quadrocopter motion is larger than the desired one, we reduce the commanded amplitude. Similarly, if the vehicle motion is lagging, we shift the commanded trajectory by increasing the phase.

4.6.1 Synchronization: The Basic Idea

To illustrate the basic idea of the "feed-forward" strategy, we consider the side-to-side motion in (4.12) and Fig. 4.6, where we adapt the commanded amplitude A_c and phase θ_c of the commanded trajectory $s_c(t)$ (Fig. 4.2),

$$x_c(t) = A_c \cos(\omega_d t + \theta_c), \quad y_c(t) = z_c(t) = \psi_c = 0. \tag{4.19}$$

to achieve synchronization. Our original results on this topic were presented in [1].

4.6.1.1 Online Correction

The motion parameters of the commanded trajectory are set to

$$\theta_c(t) = \theta_{\text{on}}(t), \quad A_c(t) = A_d + A_{\text{on}}(t), \tag{4.20}$$

where the subscript "on" indicates the online correction terms. They are updated in real time, during the flight.

As illustrated in Sect. 4.5.2, the response of the controlled quadrocopter system to a side-to-side reference signal (4.12) when choosing $s_c(t) := s_d(t)$ and $\psi_c(t) := \psi_d(t)$ is a sinusoidal signal with the same frequency but shifted phase and different amplitude,

$$x(t) = (A(t) + A_d)\ \cos(\omega_d t + \theta(t)). \tag{4.21}$$

To determine the additive errors in amplitude $A(t)$ and phase $\theta(t)$, the two reference signals, $r_{\cos}(t) = \cos \omega_d t$ and $r_{\sin}(t) = \sin \omega_d t$, are multiplied by the position estimate $x(t)$ and integrated over N periods, that is $T = (2\pi N)/\omega_d$. Assuming a constant phase shift and an amplitude error during that time interval

$$\theta(v) = \theta_t = \text{constant}, \quad A(v) = A_t = \text{constant}, \qquad t - T \le v \le t, \tag{4.22}$$

we obtain

$$\begin{aligned} \eta_1(t) &= \frac{1}{T}\int_{t-T}^{t} x(t) r_{\cos}(t) dt = \frac{A_t + A_d}{2}\cos(\theta_t), \\ \eta_2(t) &= \frac{1}{T}\int_{t-T}^{t} x(t) r_{\sin}(t) dt = -\frac{A_t + A_d}{2}\sin(\theta_t), \end{aligned} \tag{4.23}$$

and

$$\begin{aligned} A_t &= 2\sqrt{\eta_1(t)^2 + \eta_2(t)^2} - A_d, \\ \theta_t &= -\arctan\left(\eta_2(t)/\eta_1(t)\right). \end{aligned} \tag{4.24}$$

The values θ_t, A_t can be interpreted as the mean value of the phase and amplitude errors during the last period, and when considering Fig. 4.10, the phase and

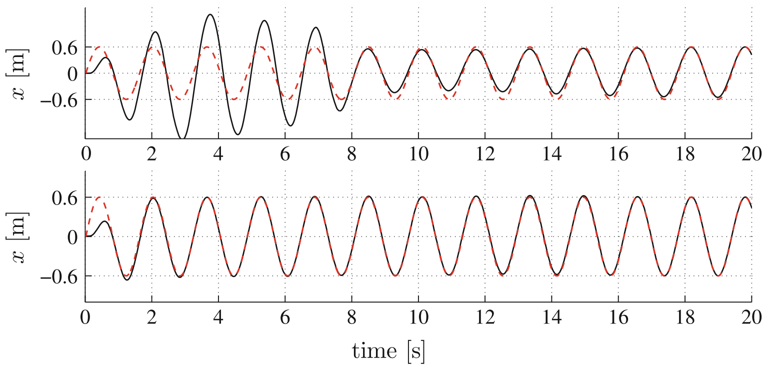

Fig. 4.12 Side-to-side motion. *Top online motion parameter adaptation only.* Quadrocopter response (*solid*) for a desired oscillation in the x direction (*dashed*) *Bottom offline motion parameter adaptation,* with online motion parameter adaptation turned on after two periods

amplitude errors are in fact constant (after a transient phase). Therefore, (4.22) is a valid assumption in steady state.

The online correction terms are calculated by integrating the errors according to

$$A_{\text{on}}(t) = k_A \int_0^t A_\tau \, d\tau, \quad \theta_{\text{on}}(t) = k_\theta \int_0^t \theta_\tau \, d\tau, \tag{4.25}$$

where the gains k_θ, k_A are chosen to ensure convergence of the online correction terms to the steady-state values $\theta_{\text{on},\infty}$ and $A_{\text{on},\infty}$, respectively.

Using the proposed online parameter adaptation strategy (4.20), (4.25), the errors in amplitude and phase are effectively regulated to zero, see Fig. 4.12 (top figure) and compare to Fig. 4.10. We observe a substantial transient phase before the online correction terms attain steady state, see Fig. 4.13. This is mainly due to the fact that the error identification scheme (4.23), (4.24) only provides reliable values after several periods.

4.6.1.2 Offline Identification

The steady-state values $\theta_{\text{on},\infty}$, $A_{\text{on},\infty}$ obtained from the online correction are repeatable (that is, different runs of the same experiment produce the same result). Consequently, the correction values can be extracted once, and later applied to improve the transient performance; that is, the tracking during the initial period of a motion. For the phase, we use

$$\theta_c(t) = \theta_{\text{off}} + \theta_{\text{on}}(t) \quad \text{with} \quad \theta_{\text{off}} = \theta_{\text{on},\infty}. \tag{4.26}$$

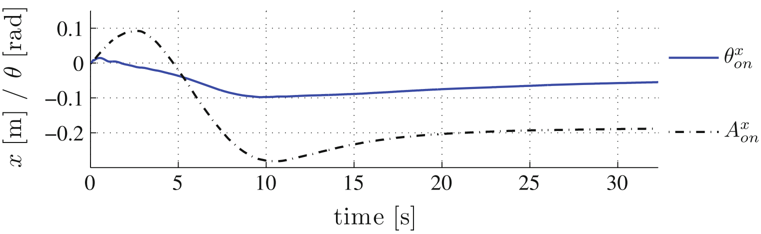

Fig. 4.13 Side-to-side motion, convergence of online correction terms

The equation for the amplitude is similar. The subscript 'off' indicates the offline motion parameters identified prior to the experiment. Figure 4.12 (bottom figure) shows the corresponding result for the side-to-side motion. The transient time is substantially decreased.

4.6.1.3 Reduced Offline Identification

Thus far, offline parameters must be identified for each side-to-side motion (A_d, ω_d) individually. To draw further conclusions, we consider the steady-state values in the following form: the amplitude-normalized amplification factor,

$$\alpha_{\text{on},\infty} = (A_d + A_{\text{on},\infty})/A_d \,, \tag{4.27}$$

and the steady-state phase $\theta_{\text{on},\infty}$ as before. Experiments in [1] have shown that the steady-state values $(\alpha_{\text{on},\infty}, \theta_{\text{on},\infty})$ depend only on the motion's frequency ω_d. That is, a single identification run must be completed for each frequency, the vehicle should perform at and the resulting parameters can be used for any side-to-side motion at this frequency with varying amplitudes.

4.6.2 Synchronization in Three Dimensions

We extend the previous results into three-dimensional (3-D) motion, which is composed of sinusoidal side-to-side motions in each translational direction:

$$\begin{bmatrix} x_d(t) \\ y_d(t) \\ z_d(t) \end{bmatrix} = \begin{bmatrix} A_d^x \cos(\omega_d^x t + \theta_d^x) \\ A_d^y \cos(\omega_d^y t + \theta_d^y) \\ A_d^z \cos(\omega_d^z t + \theta_d^z) \end{bmatrix}, \quad \psi_d(t) = 0, \tag{4.28}$$

where $\theta_d^{(x,y,z)}$ represents a potential phase shift between the sinusoidal motions in each direction. Bounces, ellipses, eights, and spirals can be obtained by appropriate parameter choices.

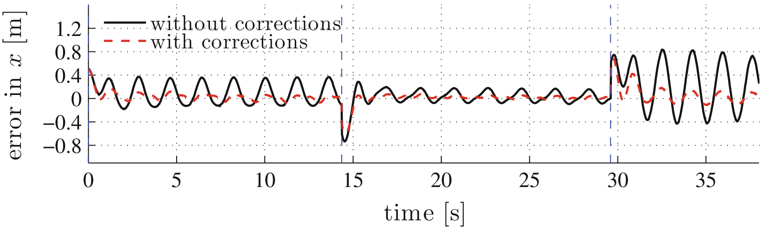

Fig. 4.14 Sequence of motions with (*dashed*) and without (*solid*) feed-forward corrections. Offline correction terms were obtained from a *reduced identification*. The errors of the desired trajectory to the response of the vehicle ($s_d(t) - s(t)$) are plotted. The motion sequence comprises: a circular motion in 3-D, a swing motion in 3-D, and a horizontal circle

As shown in [4], a key assumption can be made for the given 3-D motion: each translational direction can be treated separately. To this end, the motion parameters in the commanded trajectory $s_c(t)$ are adjusted independently for each direction according to the online strategy presented above. In addition, the offline identification benefits from the directional decoupling and the quadrocopter symmetry in that the x- and y- directions exhibit the same behavior and identification in one of the two directions is sufficient.

Consequently, it is possible to develop an identification scheme that efficiently identifies the offline correction terms for all periodic motions that can be expressed in our framework (4.28): *a single* identification run over the relevant frequency range with a 2-D motion primitive in x or y, and z is sufficient to completely identify all necessary feed-forward parameters. The offline values are stored in a look-up table ready to be used when performing new motions of (4.28).

In order to show the effectiveness of the reduced identification scheme, we perform a sequence of periodic 3-D motions with offline parameters obtained from an oscillatory motion in 2-D ($A_d^x = A_d^z = 0.4\,\text{m}$, $\omega_d^x = \omega_d^z = \omega$ and all other parameters zero). Figure 4.14 shows that the quadrocopter's deviation from the desired trajectory is clearly reduced when using the offline parameter adaptation strategy. Note that the performance can be further improved by designing smooth transitions between the motion of the sequence.

To conclude, we studied a feed-forward parameter tuning strategy that improves the tracking performance of periodic motion primitives especially during transients using preidentified correction terms and online parameter adaptation. The translational directions are independent, allowing for the efficient identification of a table stored offline. In brief, we enable quadrocopters to fly to the rhythm of the music with correctly scaled motions. This fulfills the requirements for a rhythmic motion.

4.7 Rhythmic Performances

People in the audience, when they've watched the dance, should feel like they've accomplished something, that they've gone on a journey.

—Paul Mercurio, actor and dancer

Opening night at the theater: patrons of the arts and critics take their seats while dancers do last last-minute warm-ups and take their places backstage. Tension is high: performance success is part practice, part sweat, and part luck. We have already discussed in Sect. 4.6 how quadrocopters can practice and improve their performance over time, but do sweat and luck have a role in a quadrotor performance? In control theory, the term "robustness" refers to the ability of a system to control for uncertainty; that is, for unknown effects such as wind or reduced propeller efficiencies. Given the feedback from sensors (the overhead camera system or onboard sensors), quadrocopters can react to uncertainty quickly and effectively—putting more effort into the motions if propeller efficiencies are low or executing corrective movements if unexpected external disturbances corrupt their motions. And consequently, the resulting quadrocopter performance is mostly predictable and has been demonstrated during several hundred demonstrations to visitors in and outside the lab.

4.7.1 Experimental Testbed

We demonstrate our algorithms on small, custom quadrocopters operated in the ETH Zurich Flying Machine Arena, a $10 \times 10 \times 10\,\mathrm{m}^3$ mobile testbed for quadrocopter research. The setup is similar to [51]: The system consists of a motion capture camera system that provides precise vehicle position and attitude measurements. The localization data is sent to a personal computer, which runs the control algorithms, and which in turn sends commands to the quadrocopters. More details about the test environment can be found in [52] and at: www.FlyingMachineArena.org.

4.7.2 Implementation and Robustness

Based on the rhythmic motion elements discussed in this chapter, full performances are designed for a given soundtrack. Additional motion elements not discussed in this chapter are used to smoothly concatenate the periodic motions (see [42] for more details). Moreover, acrobatic motions such as flips, loops, and bang-bang-type transitions can be incorporated in the performances to add variety; those motions are not strictly related to the music beat. In [5], the choreography design procedure is described from a practical point of view.

The resulting performance is completely preprogrammed. However, to allow for a robust and reliable execution, the preprogrammed feed-forward signals are complemented by several feedback and adaptation schemes.

We use an adaptation scheme for online synchronization of the motion to the music (Sect. 4.6). Residual phase and amplitude errors in the quadrocopter response are compensated for during the performance by adapting the commanded trajectory online (see Fig. 4.2). The online adaptation allows us to synchronize the motion of vehicles with slightly different dynamic properties (e.g., shifted center of mass and degraded propellers). The underlying trajectory-following controller (Sect. 4.5) compensates for unexpected disturbances such as wind in a reactive manner based on the measured vehicle following errors. The trajectory-following controller in turn relies on the vehicle's onboard controller to quickly compensate for local model uncertainties such as degraded propeller efficiencies by, for example, increasing the turn rate of the propellers to obtain the required thrust in the case of reduced propeller efficiencies.

4.7.3 Choreographies

Since the start of the project, several choreographies have been designed based on the rhythmic motion elements discussed in this chapter. The following list presents the choreographies that are featured in the Flying Machine Arena with the song name, the singer or composer of the song, the number of quadrocopters, and their respective design year:

- *Please don't stop the music*, Rihanna, one vehicles, 2009
- *Pirates of the Caribbean*, Hans Zimmer, two vehicles, 2009
- *Rise Up*, Yves Larock, three vehicles, 2010
- *From the Clouds*, Jack Johnson, four vehicles, 2011
- *Armageddon*, Prism, five vehicles, 2011
- *Dance of the Flying Machines*, Victor Hugo Fumagalli, six vehicles, 2013.

These choreographies have not only been regularly demonstrated at ETH Zurich, where we conduct our research, but also at exhibitions such as the Hannover Messe (April 2012, Fig. 4.15), Google I/O (June 2012), and TEDGlobal (June 2013). Figure 4.16 shows the vehicle flight trajectories that compose the first part of the *From the Clouds* performance. Associated videos are found on the project web page, www.tiny.cc/MusicInMotionSite.

4.8 Conclusions and Outlook

> *Humans do not communicate by words alone. Non-verbal behavior, including dance, is a part of the calculus of meaning.*
>
> —Judith Lynne Hanna in "To Dance is Human: A Theory of Nonverbal Communication" [53]

The evolution of robotics into human-centered applications poses important research questions, especially with respect to human–machine interaction. As long as robots remained the domain of industry, precision, speed, and repeatability were of primary importance; however, as robots increasingly enter our homes, offices, and communities, there is a corresponding need for them to be able to correctly interpret and appropriately respond to human action and behavior. In this chapter, we presented a novel visual musical experience: multiple flying vehicles coordinate their flight to the

Fig. 4.15 The ETH Zurich Flying Machine Arena at the Hannover Messe in Germany, the world's biggest industrial fair, April 2012 (*Photo* Markus Hehn)

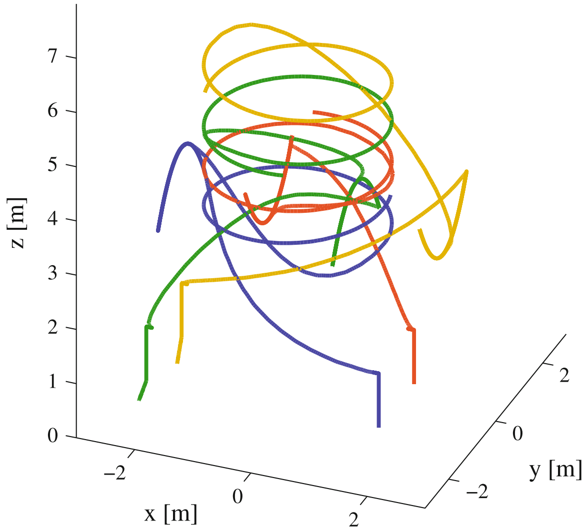

Fig. 4.16 Experimental flight data from the *From the Clouds* performance featuring four quadrocopters

rhythm of the music and perform an aerial show; a cubic indoor flight space forms the stage, and small autonomous quadrocopters are the actors of this performance. While this vision was the motivation for our work, in the process of implementing this idea, fundamental problems in trajectory planning and control were solved, including the *a priori* evaluation of a trajectory's feasibility, and a combined offline and online identification/adaptation scheme for precise tracking of periodic motions. All these components have been integrated into a software tool that facilitates the choreography design, and the range of different choreographies and the number of public demonstrations prove the feasibility and reliability of the designed algorithms. And yet the periodic motion and patterned behaviors presented here, though they may be characteristic of dance, do not in and of themselves constitute it. According to most dancers, choreographers, and audience members, it is the emotional, social, and spiritual aspects of dance that are the more essential characteristics: a robot moving rhythmically to a beat is no more a dancer than a metronome is a musical instrument. In other words, when it comes to interpreting music into a series of motions that is recognizable as dance, what is important is the *human* element.

Critical next steps for this project will thus be to explore how dance can be used as a form of shared experience with which to build an understanding of intuitive human–machine interaction. Chapter 3 explores motion-based communication in human salsa dance. As John Baillieul says, "The ultimate goal is to understand human reaction to gestures and how machines may react to gestures." Rich Barlow describes it as follows,[2] "Good dancers move seamlessly together, responding to each other's touch and motions; amateurs without experience reading each other's cues often come off looking stilted". By investigating the nonverbal cues dance partners use to communicate, the researchers hope to gain insight into new intuitive means of robot–human interaction, which could enable robots to team with, and perhaps take over from, humans in the future. Our work similarly envisions a dance "partnership" through which human–machine interaction can be studied and enhanced; however, our use of quadrocopters poses an additional challenge as there is little shared body experience between a quadrocopter body and the human body. The relationship must be founded on a shared understanding of movement alone.

Machines and humans each have their strengths and weaknesses: machines are better at rule-based, rational tasks—like synchronization and determining feasible motions sequences— whereas humans are better at things that are hard to describe using rules—like conveying and understanding emotion. Performing together as dance partners, humans and quadrocopters have the potential to engage in complimentary ways. Parallel work in our research laboratory already includes human-in-the-loop experimentation with quadrocopter control, and suggests that this kind of human–quadrocopter partnering is feasible: the TED talk www.tiny.cc/TED_DAndrea demonstrates gesture control of quadrocopters using a Kinect system, and shows simple physical interaction between a human and a quadrocopter (see Fig. 4.17 and [54]). By incorporating this research into our aerial dance system, we can enable dancers and choreographers to directly communicate with the quadrocopters using physical interaction, or simply their body language, their gestures. Imagine an experienced dancer/choreographer guiding a suite of quadrocopters as they dash through the air. What kinds of performances would we see then? Could his or her subtle touch not convey all kinds of emotion? Indeed, as described in Sect. 4.1.3, technological props have long been used to augment dance performance. However, the proposed human–quadrocopter dance "partnership" could go beyond extending a

[2] www.bu.edu/today/2013/dances-with-robots

Fig. 4.17 Demonstration of physical interaction between a human and quadrocopters: Raffaello D'Andrea at TEDGlobal, June 2013 (*Photo* James Duncan Davidson)

human's dance performance. It could ultimately help us to better understand how humans and machines can communicate intuitively with each other, and enable new forms of human–machine interaction.

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