Infinite-horizon LQR

System: $x_{k+1} = Ax_k + Bu_k \qquad k = 0, 1, \dots$

Cost: $\sum_{k=0}^{\infty} (x_k^T Q x_k + u_k^T R u_k) \quad R > 0, \; Q \geq 0$

→ given (A, B) stabilizable
(A, C) detectable with $Q = C^T C$

Then,

1) ∃ unique solution to the discrete-time algebraic Riccati equation (DARE)

$$K = A^T (K - KB(R + B^T KB)^{-1} B^T K) A + Q$$

2) the optimal feedback strategy is

$$u = Fx$$

with $F = -(R + B^T KB)^{-1} B^T KA$