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151-0563-00 **Dynamic Programming and Optimal Control** (Fall 2008)

Problem Set #5 Topic: Problems with Imperfect State Information

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The quadrocopter control more realistic...

As previously done in Problem Set #4, an infinite horizon LQR is used to control the sideways motion of a quadrocopter. However, this time, we do not assume that we have perfect state information. Actually, the only accessible state is the y position of the quadrocopter and, moreover, the control input as well as the measurement is corrupted by noise. More precisely, the quadrocopter dynamics are modelled as follows

Figure 1: Our Quadrocopter.

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w(k) \\ z(k) &= Cx(k) + v(k), \end{aligned}$$

where $w(k)$ and $v(k)$ are random variables with covariance matrix M and N, respectively. The measurement matrix C is given by $C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$. The matrices A, B, and the definition of the state $x(k)$ are as in the previous problem set. The measurement of $y(k)$ is corrupted by zero mean Gaussian noise with standard deviation of 0.01 m, and the input $u(k)$ is corrupted by zero mean Gaussian noise with standard deviation of $1\,\mathrm{rad/s^2}$.

Recalling the Separation Principle discussed in class, you can take the infinite horizon controller designed in Problem Set #4 Exercise 2.

Use a steady-state Kalman filter in order to obtain an optimal estimate of the state.

1. Write down the equations determing the closed loop dynamics of the system. Include also the estimator equations. What are the covariance matrices M and N?

2. Simulate your system assuming that your estimator is initialized to the correct value of the state. The noise can be added through the MATLAB function `randn`. Plot the input and state trajectories and show a figure comparing the real state trajectories with the estimated ones.

Additional exercises:

3. BERTSEKAS, p. 272, exercise 5.2 a)
   In addition, determine the asymptotic form of the policy as $N \to \infty$.

4. BERTSEKAS, p. 276f., exercise 5.9
   Use the hint discussed in class.

Exercises 3 to 4 are taken from the book *Dynamic Programming and Optimal Control by Dimitri P. Bertsekas, Vol. I, 3rd edition, 2005, 558 pages, hardcover.*