A Framework for Multi-Vehicle Navigation using Feedback-Based Motion Primitives

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IROS, September 25, 2017
Motivation
Motivation

Path planning and control in known environments
Problem Statement

• **Given:**
  • dynamics \( \dot{x} = f(x, u) \), outputs \( y = h(x) \), where \( x \in \mathbb{R}^n, y \in \mathbb{R}^p \)
  • goal and obstacle sets in *output space*

• **Find:** feedback controller \( u(x) \) and set of initial conditions \( X_0 \subset \mathbb{R}^n \) such that \( y(t) \) eventually enters the goal set and always avoids the obstacle set

• Can be posed as a reach-avoid problem for a control system

• Example: two double integrators, \( n = 4, p = 2 \)

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u_1 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= u_2 \\
y_1 &= x_1 \\
y_2 &= x_3
\end{align*}
\]
Framework Features

- Feedback control
  - Wide range of initial conditions
  - Robust to disturbances
  - Requires no explicit path
  - Safety guarantees
- Simultaneous motion
- Computational efficiency
  - Symmetry
  - Lower dimensional spaces
  - Modularity

Most related literature:
- Pappas
- Kumar
- Belta
- Frazzoli

\[ y(t) \]
Given

Problem Data
- $p$ outputs for multi-robot system $y = h(x)$
- obstacle and goal regions
- dynamics $\dot{x} = f(x, u)$

Output Transition System
- $p$-dim feedback control, $u_m(x)$

Maneuver Automaton
- Motion Primitives, $m_i$
- Shortest path algorithm
- discrete maneuver plan

Hybrid Control Strategy
- low-level control

Solution to problem

Automated

Partition of environment

Path planning

Control design
Maneuver Automaton

• Formally a hybrid system
• Hybrid state space: motion primitives and continuous state in $\mathbb{R}^n$
• Edges: concatenation constraints between motion primitives
• Each motion primitives is implemented by a feedback controller over a designated subset in $\mathbb{R}^n$
• First focus on double integrator: \( \dot{x}_1 = x_2, \dot{x}_2 = u \), with \( y = x_1 \)

Reach control
Maneuver Automaton - Application

• Quadrocopter model reduces to double integrator in each positional direction

• For the multi-quadrocopter model, stack all the double integrators

• Choose Hold, Forward, and Backward in each output component

• For example, one quadrocopter with planar motion:

\[
\begin{align*}
  y_1 & (H) (F) (H) (F) (B) \\
  y_2 & H H F F H \\
  y_1 & H H F F H
\end{align*}
\]

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Control Policy on the Product Automaton
Experimental Results

Two-vehicle planar motion swap places across channel
Two-vehicle planar motion swap places across channel
Two-vehicle planar motion repeat under wind disturbance
Conclusion

• Addressed a path planning and control problem in known environments as a reach-avoid problem

• Employed a modular framework consisting of an output space partition, low-level feedback controllers, and a high-level feedback for selecting motion primitives

• Highly robust control design that enables simultaneous motion in a computationally feasible way
## Comparison to Literature

<table>
<thead>
<tr>
<th>Paper</th>
<th>Feedback control</th>
<th>Simultaneous motion</th>
<th>Computational efficiency</th>
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</thead>
<tbody>
<tr>
<td>Frazzoli, Dahleh, and Feron; 2005</td>
<td>Red</td>
<td>Red</td>
<td>Green</td>
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<tr>
<td>Kloetzer and Belta; 2008</td>
<td>Green</td>
<td>Red</td>
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<td>Fainekos, Girard, Kress-Gazit, Pappas; 2009</td>
<td>Green</td>
<td>Red</td>
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<td>Ayanian, Kumar; 2010</td>
<td>Green</td>
<td>Red</td>
<td>Red</td>
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<tr>
<td>Raman, Kress-Gazit; 2014</td>
<td>Green</td>
<td>Red</td>
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<td>Vukosavljev, Kroeze, Broucke, Schoellig, 2017</td>
<td>Green</td>
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</tbody>
</table>
Stack multiple copies

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
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\dot{x}_3 &= x_4 \\
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\]

\( \dot{x} = f(x,u) \)

\( y = h(x) \)

Symmetry

Lower dimensions

Modularity