Independent vs. Joint Estimation in Multi-Agent Iterative Learning Control

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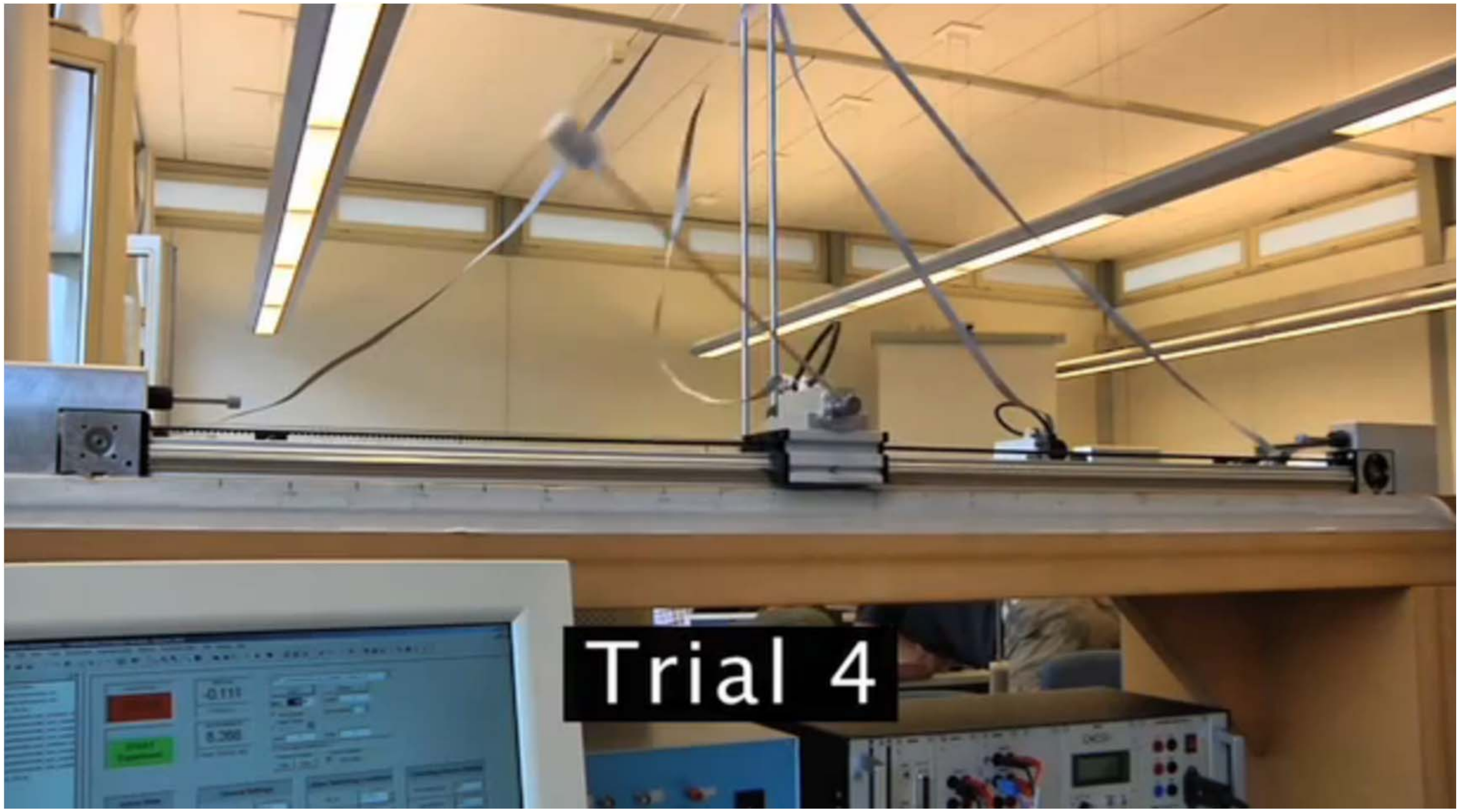
SYSTEMS ARE ABLE TO LEARN

Open-loop swing-up of a cart-pendulum system.

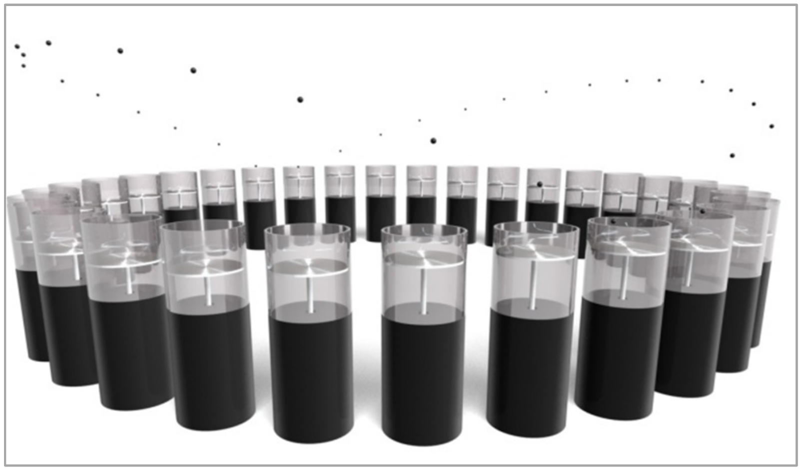
[Schöllig and D'Andrea, ECC 2009]

https://youtu.be/W2gCn6aAwz4?list=PLC12E387419CEAFF2

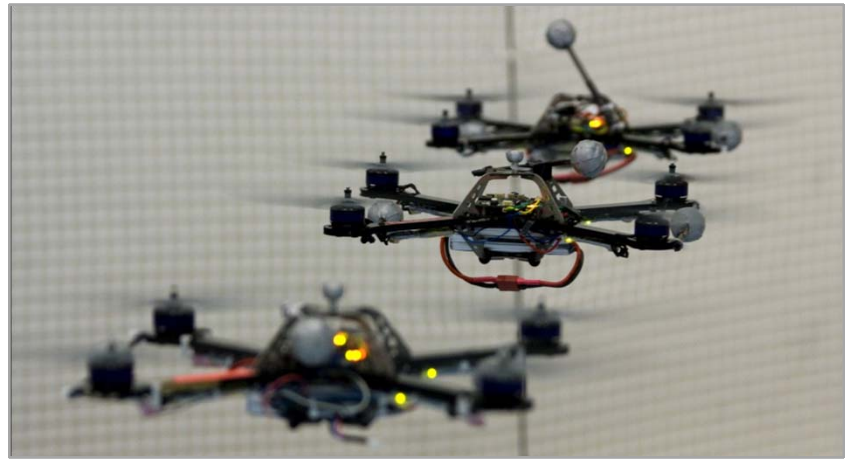


CAN SIMILAR SYSTEMS BENEFIT FROM EACH OTHER...

...when learning the same task?

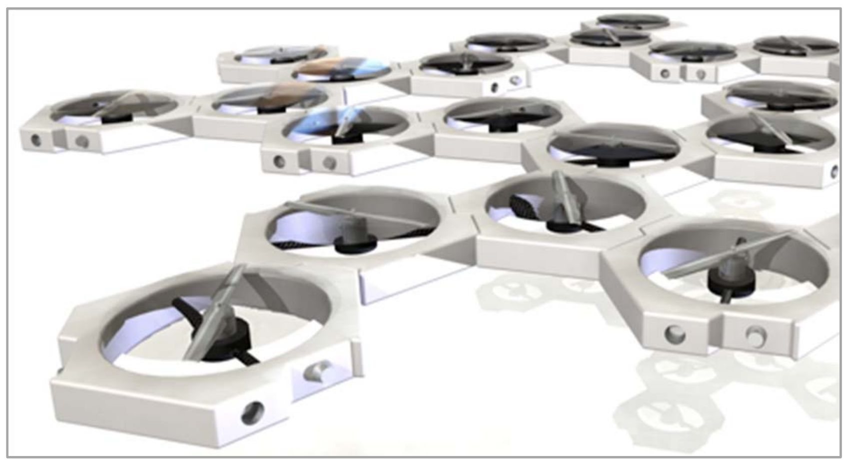


Blind Juggler Array

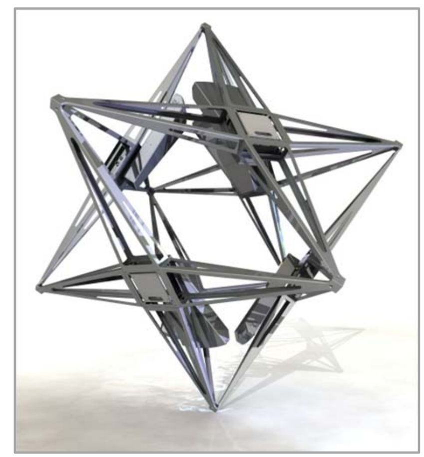


Flying Machine Arena

KIVA Systems



Distributed Flight Array

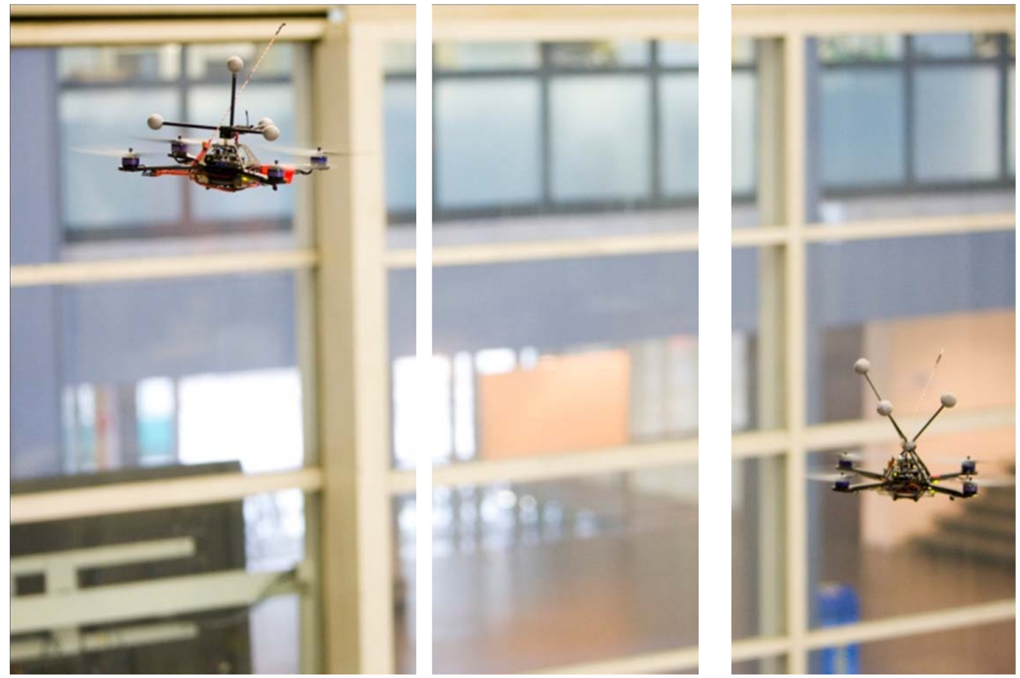


Balancing Cube

PROBLEM STATEMENT

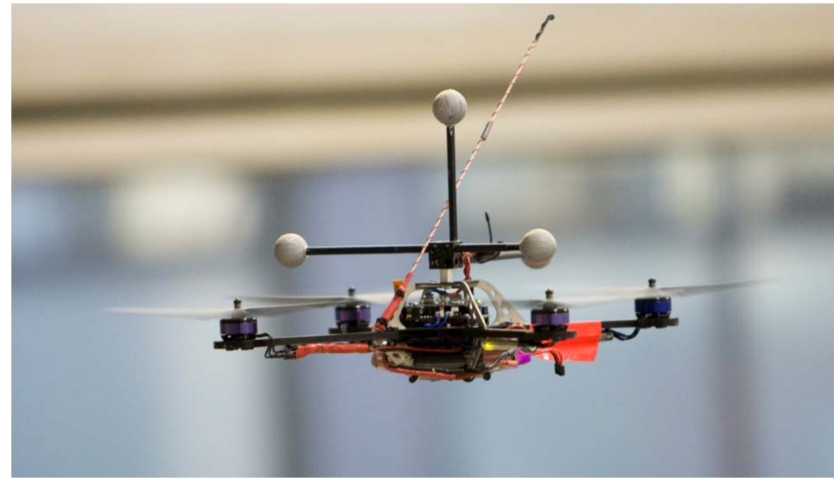
We consider

- A group of similar agents
- Performing the same task
- Repeatedly
- Simultaneous operation



Is an individual agent able to learn faster when performing a task simultaneously with a group of similar agents?

SIMILAR AGENTS (1)



Same nominal dynamics.

$$\dot{\mathrm{x}}(t) = \mathrm{f}(\mathrm{x}(t), \mathrm{u}(t))$$
$$\mathrm{y}(t) = \mathrm{g}(\mathrm{x}(t))$$

Physical model of real-world system

Same task.

$$(u^*(t),\, x^*(t),\, y^*(t))\,, \quad t \in [0,\, t_f]$$

GOAL OF LEARNING:
Follow the desired trajectory.

SIMILAR AGENTS (2)

Linearize. Small deviations from nominal trajectory.

$$\tilde{u}(t) = \mathrm{u}(t) - u^*(t), \quad \tilde{x}(t) = \mathrm{x}(t) - x^*(t), \quad \tilde{y}(t) = \mathrm{y}(t) - y^*(t)$$

Discretize. Linear, time-varying difference equations.

$$\begin{aligned} \tilde{x}(k+1) &= A_D(k)\tilde{x}(k) + B_D(k)\tilde{u}(k) \\ \tilde{y}(k) &= C_D(k)\tilde{x}(k) + D_D(k)\tilde{u}(k), \quad k \in \{0, \ldots, N\} \end{aligned}$$

Lifted-system representation. Static mapping representing one execution.

$$\underbrace{\begin{bmatrix} \tilde{x}(0) \\ \tilde{x}(1) \\ \tilde{x}(2) \\ \vdots \\ \tilde{x}(N) \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ B_D(0) & 0 & \cdots & 0 & 0 \\ \Phi_{(1,1)}B_D(0) & B_D(1) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \Phi_{(N-1,1)}B_D(0) & \Phi_{(N-1,2)}B_D(1) & \cdots & B_D(N) & 0 \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} \tilde{u}(0) \\ \tilde{u}(1) \\ \tilde{u}(2) \\ \vdots \\ \tilde{u}(N) \end{bmatrix}}_{u}$$

With $\Phi_{(l,m)} = A_D(l)A_D(l+1)\cdots A_D(m), \quad l < m$ and $\tilde{x}(0) = 0$

SIMILAR BUT NOT IDENTICAL...

In the iteration domain.

For trial $j,\ j \in \{1,\ 2,\ \ldots\}$:

$$x_j = F\,u_j + d^i + \xi_j^i$$
$$y_j = G\,x_j + \mu_j^i$$

Agent index

Measurement noise

Process noise

Iteration index

For each agent $i,\ i \in \mathcal{I} = \{1, 2, \ldots, N\}$:

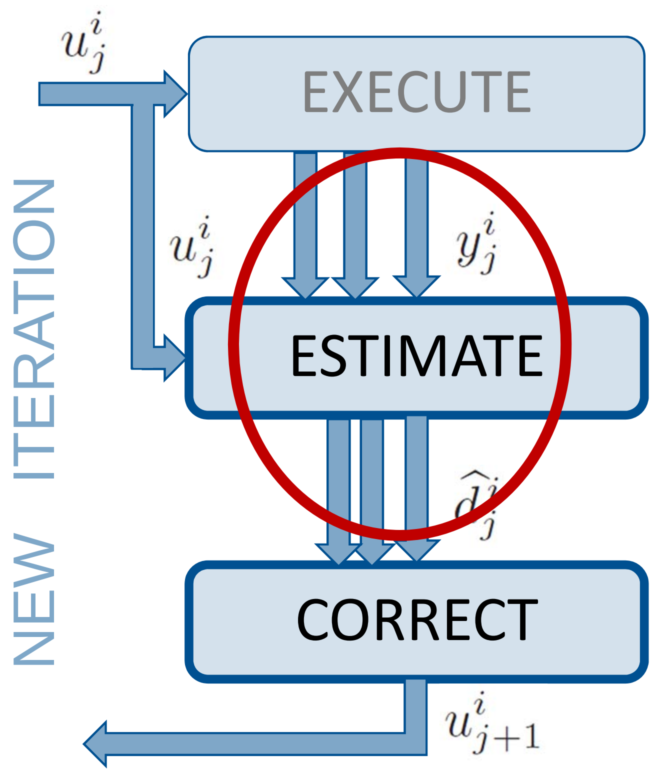
REPETITIVE DISTURBANCE

$$d^i = d^{\text{common}} + d^{i,\text{ind}}$$

Same nominal dynamics. Same task. Different repetitive disturbance.

HOW DOES A SINGLE AGENT LEARN?

$$x^i_j = F\, u^i_j + d^i + \xi^i_j$$
$$y^i_j = G\, x^i_j + \mu^i_j$$



(1) Estimate the repetitive disturbance d^i by taking into account all past measurements.

Obtain $\widehat{d}^i_j$.

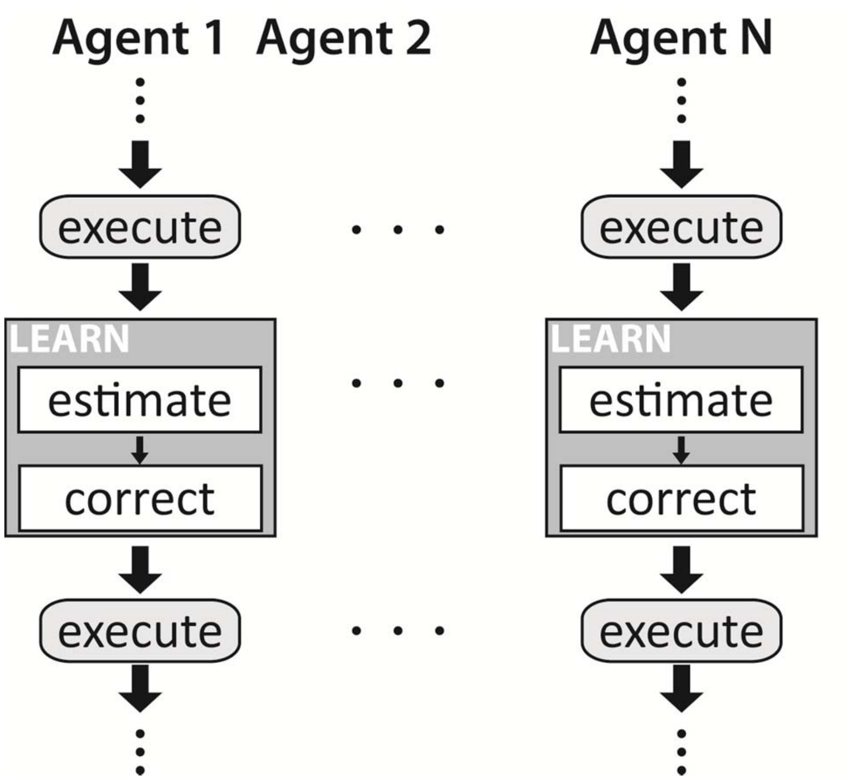
(2) Correct for $\widehat{d}^i_j$ by updating the input.

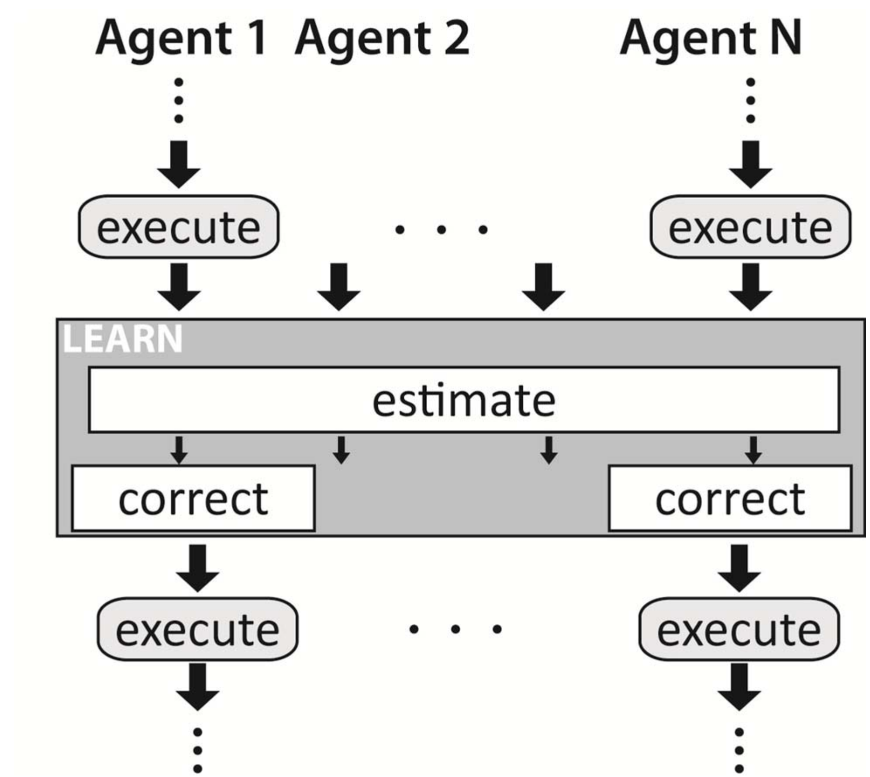
"Minimize" $x^i_{j+1} \approx F u^i_{j+1} + \widehat{d}^i_j$.

For example, $\boxed{u^i_{j+1} = \underset{u}{\operatorname{argmin}} \left\| F u + \widehat{d}^i_j \right\|}$

Can the disturbance estimate be improved by taking into account the measurements of the other agents?

FOCUS: ESTIMATION PROBLEM





INDEPENDENT ESTIMATION vs. JOINT ESTIMATION

REDUCE MODEL

DYNAMICS

$$x_j^i = \cancel{F x_j^i} + d^i + \xi_j^i$$
$$y_j^i = \cancel{G}\, x_j^i + \mu_j^i$$

- neglect deterministic part
- assume state is measured directly
- assume independence and same noise characteristics for vector entries

$$y_j^i = d^{\text{common}} + d^{i,\text{ind}} + v_j^i \in \mathbb{R}$$

with
$$\begin{aligned} d^{\text{common}} &\sim \mathcal{N}(0, \sigma^{\text{common}}) \\ d^{i,\text{ind}} &\sim \mathcal{N}(0, \sigma^{\text{ind}}) \\ v_j^i &\sim \mathcal{N}(0, 1) \end{aligned}$$

MEASUREMENT AND PROCESS NOISE
$$v_j^i = \xi_j^i + \mu_j^i$$

LEARNING PERFORMANCE is measured by the variance of the state estimate.

JOINT ESTIMATION

Estimation objective.

$$D = [\, d^{\text{common}},\, d^1,\, \dots,\, d^N \,]^{\mathrm{T}} \;\in \mathbb{R}^{N+1}$$

Kalman equations.

$$\begin{aligned} D_j &= D_{j-1} \\ Y_j &= [\,\mathbf{0},\ I\,]\ D_j + V_j \end{aligned}$$

Variance of disturbance estimate.

$$P_j = [\, p_j^{(k,l)} \,]$$

PROPOSITION: Covariance of an individual's disturbnance estimate

$$p_j^{(1,1)} \quad = \frac{\sigma^{\text{common}} + \sigma^{\text{ind}} + j\left(\sigma^{\text{ind}}\right)^2 + jN\sigma^{\text{common}}\sigma^{\text{ind}}}{(1 + j\sigma^{\text{ind}})(1 + j\sigma^{\text{ind}} + jN\sigma^{\text{common}})}$$

INDEPENDENT CASE: $\left. p_j^{(1,1)} \right|_{N=1}$

COMPARISON

$$x_j^i = d^i + \xi_j^i$$
$$y_j^i = x_j^i + \mu_j^i$$

COVARIANCE OF STATE ESTIMATE:

$$E\left[(x_j^i - \widehat{x}_j^i)^2\right] = E\left[(d^i + \xi_j^i - \widehat{d}_j^i)^2\right] \quad \text{with} \quad \widehat{x}_j^i = \widehat{d}_j^i$$

$$= p_j^{(1,1)} + \mathrm{Var}\left(\xi_j^i\right) .$$

RATIO OF COVARIANCE: independent vs. joint estimation

(I) PURE PROCESS NOISE

$$R^{\text{proc}} = \frac{p_j^{(1,1)}\big|_{N=1} + 1}{p_j^{(1,1)} \qquad + 1}$$

(II) PURE MEASUREMENT NOISE

$$R^{\text{meas}} = \frac{p_j^{(1,1)}\big|_{N=1}}{p_j^{(1,1)}}$$

RESULT

$$d^{\text{common}} \sim \mathcal{N}(0,\, \sigma^{\text{common}})$$
$$d^{i,\text{ind}} \sim \mathcal{N}(0,\, \sigma^{\text{ind}})$$

Performance increase due to joint estimation:

THEOREM 1: Pure Process Noise

$$1 \leq R^{\text{proc}} \leq \frac{1+j}{j} \qquad \forall\, \sigma^{\text{common}},\, \sigma^{\text{ind}},\, N,\, j$$

limit case for $N \to \infty,\ \sigma^{\text{common}} \to \infty,\ \sigma^{\text{ind}} \to 0$

THEOREM 2: Pure Measurement Noise

$$1 \leq R^{\text{meas}} \leq N \qquad \forall\, \sigma^{\text{common}},\, \sigma^{\text{ind}},\, N,\, j$$

limit case for $\sigma^{\text{common}} \to \infty,\ \sigma^{\text{ind}} \to 0$

EXAMPLE

For 10 agents:

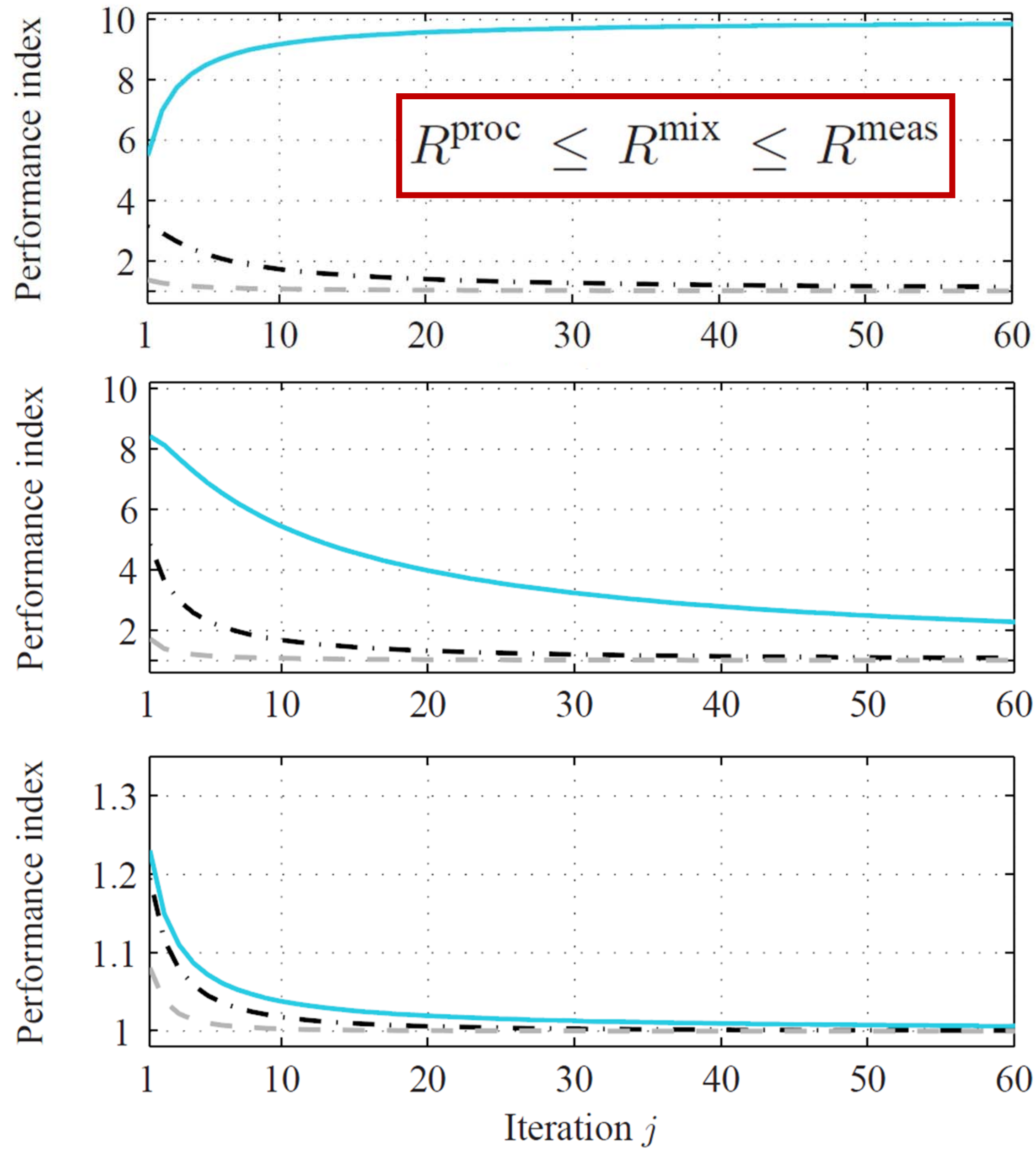
$$\sigma^{\text{common}} = 1,\ \sigma^{\text{ind}} = 0$$

$$\sigma^{\text{common}} = 10,\ \sigma^{\text{ind}} = 0.01$$

$$\sigma^{\text{common}} = 1,\ \sigma^{\text{ind}} = 1$$

$$1 \leq R^{\text{proc}} \leq \frac{1+j}{j}$$

$$1 \leq R^{\text{meas}} \leq N$$



JOINT ESTIMATION IS ONLY BENEFICIAL IF...

(1) High similarity between agents

(2) Process noise negligible

(3) Common model error large compared to the noise

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