Independent vs. Joint Estimation in Multi-Agent Iterative Learning Control

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SYSTEMS ARE ABLE TO LEARN

Open-loop swing-up of a cart-pendulum system.
[Schöllig and D'Andrea, ECC 2009]

https://youtu.be/W2gCn6aAwz4?list=PLC12E387419CEAFF2
CAN SIMILAR SYSTEMS BENEFIT FROM EACH OTHER...

...when learning the same task?
PROBLEM STATEMENT

We consider

• A group of similar agents
• Performing the same task
• Repeatedly
• Simultaneous operation

Is an individual agent able to learn faster when performing a task simultaneously with a group of similar agents?
SAME NOMINAL DYNAMICS.

\[
\dot{x}(t) = f(x(t), u(t)) \\
y(t) = g(x(t))
\]

SAME TASK.

\[(u^*(t), x^*(t), y^*(t)), \quad t \in [0, t_f]\]

GOAL OF LEARNING: Follow the desired trajectory.
SIMILAR AGENTS (2)

**Linearize.** Small deviations from nominal trajectory.

\[ \tilde{u}(t) = u(t) - u^*(t), \quad \tilde{x}(t) = x(t) - x^*(t), \quad \tilde{y}(t) = y(t) - y^*(t) \]

**Discretize.** Linear, time-varying difference equations.

\[ \tilde{x}(k + 1) = A_D(k)\tilde{x}(k) + B_D(k)\tilde{u}(k) \]
\[ \tilde{y}(k) = C_D(k)\tilde{x}(k) + D_D(k)\tilde{u}(k), \quad k \in \{0, \ldots, N\} \]

**Lifted-system representation.** Static mapping representing one execution.

\[
\begin{bmatrix}
\tilde{x}(0) \\
\tilde{x}(1) \\
\tilde{x}(2) \\
\vdots \\
\tilde{x}(N)
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & \cdots & 0 & 0 \\
B_D(0) & 0 & \cdots & 0 & 0 \\
\Phi_{(1,1)}B_D(0) & B_D(1) & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\Phi_{(N-1,1)}B_D(0) & \Phi_{(N-1,2)}B_D(1) & \cdots & B_D(N) & 0
\end{bmatrix} 
\begin{bmatrix}
\tilde{u}(0) \\
\tilde{u}(1) \\
\tilde{u}(2) \\
\vdots \\
\tilde{u}(N)
\end{bmatrix}
\]

With \( \Phi_{(l,m)} = A_D(l)A_D(l+1)\cdots A_D(m), \ l < m \) and \( \tilde{x}(0) = 0 \)

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In the iteration domain.

For trial \( j, j \in \{1, 2, \ldots \} \):

\[
x_j = F u_j + d^i + \xi_j
\]

\[
y_j = G x_j + \mu_j
\]

For each agent \( i, i \in \mathcal{I} = \{1, 2, \ldots, N\} \):

\[
d^i = d^\text{common} + d^i,\text{ind}
\]

Same nominal dynamics. Same task. Different repetitive disturbance.
Estimate the repetitive disturbance $d^i$ by taking into account all past measurements. Obtain $\hat{d}_j^i$.

Correct for $\hat{d}_j^i$ by updating the input. “Minimize” $x_{j+1}^i \approx F u_{j+1}^i + \hat{d}_j^i$.

Can the disturbance estimate be improved by taking into account the measurements of the other agents?

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FOCUS: ESTIMATION PROBLEM

INDEPENDENT ESTIMATION vs. JOINT ESTIMATION

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REDUCE MODEL

DYNAMICS

- neglect deterministic part
- assume state is measured directly
- assume independence and same noise characteristics for vector entries

\[
\begin{align*}
  x_j^i &= F x_j^i + d^i + \xi_j^i \\
  y_j^i &= \epsilon x_j^i + \mu_j^i
\end{align*}
\]

\[
y_j^i = d_{\text{common}}^i + d_{\text{ind}}^i + \nu_j^i \in \mathbb{R}
\]

with

\[
\begin{align*}
  d_{\text{common}}^i &\sim \mathcal{N}(0, \sigma_{\text{common}}) \\
  d_{\text{ind}}^i &\sim \mathcal{N}(0, \sigma_{\text{ind}}) \\
  \nu_j^i &\sim \mathcal{N}(0, 1)
\end{align*}
\]

MEASUREMENT AND PROCESS NOISE

\[
\nu_j^i = \xi_j^i + \mu_j^i
\]

LEARNING PERFORMANCE is measured by the variance of the state estimate.

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JOINT ESTIMATION

Estimation objective.

\[ D = [d^\text{common}, d^1, \ldots, d^N]^T \in \mathbb{R}^{N+1} \]

Kalman equations.

\[ D_j = D_{j-1} \]
\[ Y_j = [0, I] D_j + V_j \]

Variance of disturbance estimate.

\[ P_j = [p_j^{(k,l)}] \]

PROPOSITION: Covariance of an individual’s disturbance estimate

\[ p_{j}^{(1,1)} = \frac{\sigma^\text{common} + \sigma^\text{ind} + j \left(\sigma^\text{ind}\right)^2 + jN\sigma^\text{common} \sigma^\text{ind}}{(1 + j\sigma^\text{ind})(1 + j\sigma^\text{ind} + jN\sigma^\text{common})} \]

INDEPENDENT CASE: \[ p_{j}^{(1,1)} \bigg|_{N=1} \]

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COMPARISON

COVARIANCE OF STATE ESTIMATE:

\[
E \left[ (x^i_j - \hat{x}^i_j)^2 \right] = E \left[ (d^i + \xi^i_j - \hat{d}^i_j)^2 \right] \quad \text{with} \quad \hat{x}^i_j = \hat{d}^i_j \\
= p_j^{(1,1)} + \text{Var} (\xi^i_j) .
\]

RATIO OF COVARIANCE: independent vs. joint estimation

(I) PURE PROCESS NOISE

\[
R^{\text{proc}} = \frac{p_j^{(1,1)} \bigg|_{N=1}}{p_j^{(1,1)}} + 1
\]

(II) PURE MEASUREMENT NOISE

\[
R^{\text{meas}} = \frac{p_j^{(1,1)} \bigg|_{N=1}}{p_j^{(1,1)}}
\]

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RESULT

Performance increase due to joint estimation:

THEOREM 1: Pure Process Noise

\[ 1 \leq R^{\text{proc}} \leq \frac{1 + j}{j} \]

\( \forall \sigma^{\text{common}}, \sigma^{\text{ind}}, N, j \)

limit case for \( N \to \infty, \sigma^{\text{common}} \to \infty, \sigma^{\text{ind}} \to 0 \)

THEOREM 2: Pure Measurement Noise

\[ 1 \leq R^{\text{meas}} \leq N \]

\( \forall \sigma^{\text{common}}, \sigma^{\text{ind}}, N, j \)

limit case for \( \sigma^{\text{common}} \to \infty, \sigma^{\text{ind}} \to 0 \)
EXAMPLE

For 10 agents:

\[ \sigma_{\text{common}} = 1, \ \sigma_{\text{ind}} = 0 \]

\[ \sigma_{\text{common}} = 10, \ \sigma_{\text{ind}} = 0.01 \]

\[ 1 \leq R^{\text{proc}} \leq \frac{1 + j}{j} \]

\[ 1 \leq R^{\text{meas}} \leq N \]
JOINT ESTIMATION IS ONLY BENEFICIAL IF...

(1) High similarity between agents

(2) Process noise negligible

(3) Common model error large compared to the noise
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