


Deep Neural Networks as Add-on Modules for Enhancing Robot Performance in Impromptu Trajectory Tracking

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Abstract

High-accuracy trajectory tracking is critical to many robotic applications, including search and rescue, advanced manufacturing, and industrial inspection, to name a few. Yet the unmodeled dynamics and parametric uncertainties of operating in such complex environments make it difficult to design controllers that are capable of accurately tracking arbitrary, feasible trajectories from the first attempt (i.e., impromptu trajectory tracking). This paper proposes a platform-independent, learning-based “add-on” module to enhance the tracking performance of black-box control systems in impromptu tracking tasks. Our approach is to pre-cascade a deep neural network (DNN) to a stabilized baseline control system, in order to establish an identity mapping from the desired output to the actual output. Previous research involving quadrotors showed that, for 30 arbitrary hand-drawn trajectories, the DNN-enhancement control architecture reduces tracking errors by 43% on average, as compared to the baseline controller. In this paper, we provide a platform-independent formulation and practical design guidelines for the DNN-enhancement approach. In particular, we: 1) characterize the underlying function of the DNN module; 2) identify necessary conditions for the approach to be effective; 3) provide theoretical insights into the stability of the overall DNN-enhancement control architecture; 4) derive a condition that supports data-efficient training of the DNN module; and 5) compare the novel theory-driven DNN design with the prior trial-and-error design using detailed quadrotor experiments. We show that, as compared to the prior trial-and-error design, the novel theory-driven design allows us to reduce the input dimension of the DNN by two thirds while achieving similar tracking performance.

Keywords

Motion control, Learning and adaptive systems, Aerial robotics, Trajectory tracking, Neural networks

1 Introduction

As continued advancements in algorithms, actuation, and sensor technology push robots into more complex environments, increasingly sophisticated methods for controlling robot motion are needed. In particular, controllers that are capable of high-accuracy trajectory tracking are becoming increasingly important in robot applications where safety and/or efficiency are essential; for example: in search and rescue, where robots must operate in close proximity to people (Liu and Nejat 2013); in advanced manufacturing, where robot arms must efficiently follow pre-designed trajectories to perform complex manipulation tasks (Brogårdh 2007); or in industrial inspection, where unmanned aerial vehicles fly in close proximity to facilities to enable visual inspection (Nikolic et al. 2013).

The trajectory tracking problem has been extensively studied in the control literature. Among various techniques, the proportional-integral-derivative (PID) controller is often used in trajectory tracking applications. However, tuning PID parameters is typically time-consuming, and the performance of a PID controller can be conservative (Åström and Hägglund 2004). Moreover, control theory shows that a standard PID control architecture cannot achieve exact tracking for arbitrary trajectories (Francis and Wonham 1976).

In addition to the PID controller, model-based techniques such as Model Predictive Control (MPC) have been studied for finding optimal control commands that lead to accurate and agile robot motions (Liniger et al. 2015). Moreover, inversion-based feedforward approaches have been widely applied to achieve high-accuracy tracking (Hirschorn 1979; Devasia et al. 1996). However, one general limitation of model-based approaches is the reliance on a sufficiently accurate dynamic model of the system, which is difficult to obtain in practice. Adaptive control (Slotine and Li 1987) and robust control (Spong 1992) strategies have been used to address uncertainties in system parameters. Yet while these approaches typically guarantee stability, they do not take past experience into account, and the same errors are repeated from trial to trial if the same reference is given.

As robot dynamics and operating environments become ever more complex, researchers are increasingly turning to learning-based approaches to address the resulting model

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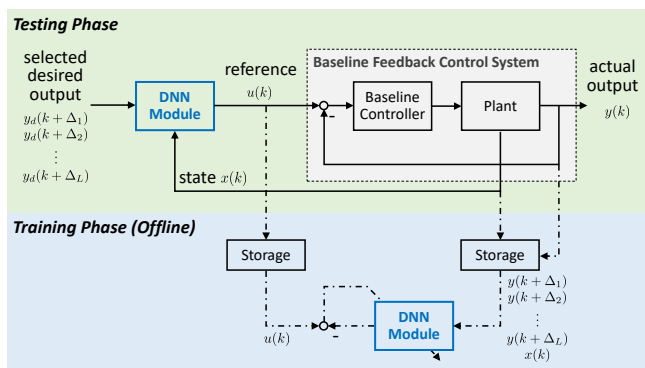


Fig. 1. The DNN-enhancement control architecture: During the training phase (blue rectangle), a baseline system is treated as a black-box and the baseline system input, output, and state data are collected for training a DNN module. During the testing phase (green rectangle), the DNN module, as an add-on component to the baseline system, is pre-cascaded to the baseline system and adjusts the reference signals to enhance the baseline system performance.

uncertainties. These learning-based approaches have been successfully applied to manipulators (Levine et al. 2015), bipedal robots (Da et al. 2017), autonomous cars (Drews et al. 2017), and unmanned aerial vehicles (Tang and Kumar 2018; Bansal et al. 2016), to name a few. A common learning-based approach that yields high-accuracy tracking is iterative learning control (ILC). In ILC, the tracking performance is improved by adjusting control inputs or reference signals in repeated trials (Bristow et al. 2006; Schoellig et al. 2012; Tayebi 2004). In addition to ILC, reinforcement learning (RL)-based approaches have also been proposed to iteratively optimize the tracking performance (Kiumarsi et al. 2014; Zhang et al. 2016a; Pane et al. 2016). Apart from iterative approaches, there are also various works on improving the tracking performance of classical model-based controllers by learning the uncertain or unknown system dynamics with techniques such as Gaussian Processes (GPs) (Nguyen-Tuong and Peters 2008; Helwa et al. 2018), Neural Networks (NNs) (Yan and Wang 2014; He et al. 2016), and Support Vector Machines (SVMs) (Iplikci 2006). Alternatively, these learning techniques have also been applied to improve the tracking performance by approximating inverse dynamic models in inversion-based feedforward approaches (Nguyen-Tuong and Peters 2010; Schaal et al. 2002).

In this paper, we consider the impromptu tracking problem. That is, we aim to achieve high-accuracy tracking of arbitrary, feasible trajectories from the first attempt. Motivated by the success of the learning-based control approaches for robot control, we present a deep neural network (DNN)-based approach for enhancing the impromptu tracking control performance of black-box systems. This paper extends our previous work (Li et al. 2017), in which a DNN add-on module was used to improve the performance of quadrotors in tracking arbitrary, hand-drawn trajectories. The proposed DNN-enhancement architecture is illustrated in Fig. 1. During the training phase, the input, output, and state of the baseline system are recorded for training a DNN module. Then, during the testing phase, the DNN module is pre-cascaded to the

baseline system to adapt the reference signals to establish an identity mapping from the desired output to the actual output. In (Li et al. 2017), experiments on 30 arbitrary, hand-drawn trajectories show that the DNN-enhancement control architecture effectively reduces the tracking error of the quadrotor vehicle by 43% on average as compared to the baseline controller. As compared with the other learning-based tracking control approaches, the proposed DNN-approach has the following advantages:

- Unlike the iterative learning methods (ILC approaches and some RL-based approaches such as (Pane et al. 2016)), the proposed DNN approach can be directly used for tracking arbitrary, feasible trajectories without further adaptations during the testing phase, and consequently, it satisfies the impromptu tracking requirement.
- Compared to more common approaches (such as forward or inverse dynamic learning) where the learning component typically resides in the main control loop, we use the DNN module as an add-on block that is placed outside of the closed-loop system to improve the tracking performance. This add-on approach enables black-box control systems to be improved retrospectively.
- As will be discussed in the following section, the proposed approach is less prone to instability than other inverse-based approaches because the DNN loop can be run at a lower rate than the baseline control loop (Li et al. 2017). Moreover, the proposed architecture can potentially lead to better learning-enhanced performance as the closed-loop system has a more repeatable behaviour (Mueller et al. 2012).

While experiments from (Li et al. 2017) have shown that the DNN approach shown in Fig. 1 is effective for quadrotors, the DNN module was designed by trial-and-error, and guidelines for systematically applying the approach to other robotic platforms were not given. In this work, we develop a platform-independent formulation of the DNN approach. The proposed formulation includes characterizing the underlying function learned by the DNN module, identifying necessary conditions for the DNN approach to be effective, and, with a practical incentive, deriving a condition that allows the training of the DNN module to be more efficient. In (Zhou et al. 2017) we presented a preliminary version of the DNN add-on concept, and discussed the application of this approach to minimum phase systems (i.e., systems with stable inverse dynamics); in (Zhou et al. 2018) application of the approach to nonminimum phase systems (i.e., systems with unstable inverse dynamics) is discussed. In the current paper we focus on minimum phase systems, and extend the main theoretical insights of (Zhou et al. 2017) from single-input-single-output (SISO) systems to multi-input-multi-output (MIMO) systems. We also provide a proof of the control system's overall stability, and present significantly more simulations and experiments to support the theoretical extensions.

Note that, in the first work (Li et al. 2017), DNN is chosen as the learning technique to construct the add-on block. This design decision was motivated by the fact that the

amount of memory and computational cost of the forward pass of the DNN is fixed as more data is collected. In contrast to nonlinear regression methods such as GP, the relatively fixed memory and computational cost allow the DNN model to be implemented on robot platforms where onboard computational resources are limited (Li et al. 2017). Following (Li et al. 2017), we use DNN as the learning technique in this work; however, the presented theoretical insights can be potentially generalized to other nonlinear regression techniques.

2 Related Work on NN-Based Inverse Control

The DNN module in the proposed control architecture (Fig. 1) aims to establish an identity mapping from the desired output to the actual output (Li et al. 2017). In the literature of NN-based control, common approaches that have a similar objective include *direct inverse control*, *feedback-error learning control*, and *adaptive inverse control*.

In direct inverse control, an NN is trained to approximate the inverse dynamics of the open-loop plant, and is pre-cascaded to the plant as the controller to achieve exact tracking (Hunt et al. 1992; Suprijono et al. 2015). Early literature such as (Jordan and Rumelhart 1992; Kawato 1990) compared different approaches for training the NN inverse model and discussed practical implementation concerns. For example, (Jordan and Rumelhart 1992) pointed out that an NN directly trained with the reversed input-output data from the open-loop plant is not ‘goal-directed’ — the training objective of minimizing the regression error of the model output does not directly reflect the control objective of minimizing the tracking error of the system. To address these concerns, training schemes such as the distal teacher (Jordan and Rumelhart 1992) have been proposed. However, apart from these discussions, a fundamental drawback of the direct inverse control approach is the lack of robustness against disturbances in the system. This drawback is attributed to the fact that the NN inverse model is often used as the only controller of the system.

To address the issues with direct inverse control, (Kawato 1990) proposes a feedback-error learning scheme. This approach employs a feedback control loop, where the input command to the plant is the sum of the signal from the feedback controller and feedforward signal from an NN-based inverse model. In contrast with typical direct inverse control, the error signal for training the NN is the output of the feedback controller instead of the typical regression errors based on the plant input-output data. Although practical considerations such as the ‘goal-directness’ issue and robustness issue are addressed in the feedback error learning approach, the training of the NN requires a plant in the loop, which may not be desired in the early training phase.

Another inversion-based approach for trajectory tracking problems is adaptive inverse control, in which the parameters of an NN controller are updated online with guaranteed stability (Chen and Khalil 1995; Ge and Zhang 2003; Zhang et al. 2016b). A limitation of the adaptive-NN approach is that the number of adaptive parameters, and

hence the online computational cost, can be large. Moreover, good NN parameter initializations are typically needed for convergence (Chen and Khalil 1995).

Overall, despite the similarity in the control objective, there are fundamental differences between the proposed DNN control architecture in Fig. 1 and the common NN-based inverse control architectures. One of the differences is that the proposed DNN control architecture modifies the reference of a stabilized closed-loop system, while the common NN-based inverse control approaches directly modify the input to the open-loop plant. From a practical perspective, this difference has two potential benefits: (i) By introducing the DNN as an outer loop that runs at a lower rate as compared to the baseline system, the overall approach is less prone to stability issues (Li et al. 2017). (ii) Since the closed-loop system partially compensates non-repeated disturbances, the response of the closed-loop system is more repeatable than that of the open-loop plant (Mueller et al. 2012). Thus, learning to adapt the reference of a closed-loop system can be potentially more effective for achieving good tracking performance. In contrast to adaptive inverse control (in which high-accuracy tracking control and stability of the plant are simultaneously achieved by the designed NN parameter update laws), the proposed DNN approach achieves stabilization through the design of the baseline controller, and tracking performance is enhanced separately by the pre-cascaded DNN module. This approach of decoupling the stabilization and tracking performance enhancement problems can greatly simplify the DNN design and training in practical applications. We furthermore investigate the effectiveness of the proposed DNN approach for the problem of impromptu tracking, and verify this experimentally by testing whether quadrotors are able to accurately fly arbitrary, hand-drawn trajectories from the first attempt. Although the NN-based inverse control approaches in the literature provide theoretical foundations for designing high-accuracy tracking controllers, their ability to track arbitrary, feasible trajectories has not been thoroughly demonstrated in experiments.

3 Problem Formulation

Our objective is to enhance black-box control systems to achieve high-accuracy, impromptu tracking. In previous work (Li et al. 2017), with quadrotors as the test platform, a DNN-enhancement control architecture (Fig. 1) is proposed to establish an identity mapping from the desired output y_d to the actual output y . In this work, we aim to provide a platform-independent formulation of the proposed DNN-enhancement control architecture (Li et al. 2017). This formulation includes:

- (O1) identifying the underlying function that should be represented by the DNN module in order to establish an identity mapping from y_d to y ;
- (O2) identifying necessary conditions for the approach to be effective;
- (O3) deriving guidelines for systematically selecting the inputs and outputs of the DNN module;

- (O4) analyzing the stability of the DNN-enhanced system in the presence of regression errors; and
- (O5) characterizing a condition that allows for further improving data efficiency of the DNN training.

In the following discussion, we first consider linear time invariant (LTI) multi-input-multi-output (MIMO) baseline systems represented by the following state space model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k), \end{aligned} \quad (1)$$

where $k \in \mathbb{Z}_{\geq 0}$ denotes the discrete-time index, $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the system input, $y \in \mathbb{R}^m$ is the system output, and A , B , and C are constant matrices of appropriate dimensions. After presenting the insights from the linear system formulation, we then extend the discussion to nonlinear MIMO baseline systems represented by

$$\begin{aligned} x(k+1) &= f(x(k)) + g(x(k))u(k), \\ y(k) &= h(x(k)), \end{aligned} \quad (2)$$

where $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ are matrices of smooth functions with appropriate dimensions. Note that, in the discussion of this work, we focus on square MIMO systems having the same number of inputs and outputs. This is not a restrictive formulation for tracking applications, since systems (1) and (2) typically represent baseline closed-loop systems, and each output in y has a corresponding reference input in u .

In deriving the theoretical insights for this paper, we make the following assumptions:

- (A1) the baseline system is input-to-state stable (Jiang and Wang 2001). For the nonlinear system (2), we additionally assume that the state can be bounded by

$$\|x\|_{\infty} \leq L_1 \|u\|_{\infty} + L_2 \|x_0\| + L_3, \quad (3)$$

where $\|\cdot\|_{\infty}$ denotes the infinity norm, $x_0 \in \mathbb{R}^n$ is the initial state, and L_1 , L_2 , and L_3 are constant, positive scalars;

- (A2) at any instant k , a preview of n future time steps of the desired trajectory (i.e., $\{y_d(k), y_d(k+1), \dots, y_d(k+n)\}$) is available, where n is the system order;
- (A3) the DNN module has a feedforward architecture and globally Lipschitz activation functions.

Note that assumptions (A1)-(A3) are not restrictive. Assumption (A1) on the stability of the baseline closed-loop system can be achieved by proper controller designs with well-developed control techniques even in the absence of a detailed or highly-accurate dynamic model of the system. The inequality (3) in Assumption (A1) holds for input-to-state stable linear systems; for nonlinear systems, this is an additional assumption that we use to provide a theoretical guarantee on stability of the overall control system. For assumption (A2), a preview of n steps of the desired trajectory is usually available in practice, and does not prevent combinations with online trajectory generation algorithms. For assumption (A3), although we use feedforward neural networks (FNNs) in this work,

the proposed approach can be potentially adapted for use with other nonlinear regression techniques (e.g., GPs, recurrent neural networks). The globally Lipschitz condition in assumption (A3) holds for the commonly-used activation functions such as the rectified linear units (ReLU), sigmoid, and hyperbolic tangent.

4 Theoretical Insights

In this section we provide four theoretical insights to achieve the objectives (O1)-(O5) stated in Sec. 3. We begin our discussion with a background on the inversion of dynamic systems in Sec. 4.1. We then build on this conceptual overview in Sec. 4.2 to derive the underlying function to be modeled by the DNN module to establish an identity mapping between the desired and actual outputs, and identify conditions that are necessary for the proposed DNN approach to be effective. Building on the insight regarding the underlying function modeled by the DNN module, we provide guidelines for systematically selecting the inputs and outputs of the DNN module in Sec. 4.3, provide a proof of stability of the overall control system in the presence of DNN regression errors in Sec. 4.4, and derive a condition that allows us to further improve the data-efficiency of the DNN training in Sec. 4.5.

4.1 Background on System Inversion

Starting with the DNN-enhancement control architecture in Fig. 1, (Li et al. 2017) initially designed a DNN module with $y_d(k)$ and $x(k)$ as input and $u(k)$ as output to enhance the tracking performance of the quadrotor baseline control system. The experiments of (Li et al. 2017) show that the DNN module is able to enhance the tracking performance of the baseline system only after $y_d(k)$ in the DNN input is replaced by certain future desired outputs $\{y_d(k+\Delta_1), y_d(k+\Delta_2), \dots, y_d(k+\Delta_L)\}$ with $\Delta_1, \Delta_2, \dots, \Delta_L > 0$ selected based on trial-and-error. As will be shown in Sec. 4.2, this experimental observation can be explained by associating the DNN module with the inverse dynamics of the baseline system.

In order to facilitate the following discussions, in this subsection, we state the formal definition of the *vector relative degree* (Jang et al. 1994; Isidori 1995), and discuss its connection to the system inverse. In the discussions below, we use $h \circ f$ to denote the composition of the functions h and f , and f^i to denote the i -th composition of the function f with $f^0(x(k)) = x(k)$ and $f^i(x(k)) = f \circ f^{i-1}(x(k))$.

Definition 4.1. Vector Relative Degree. *The nonlinear MIMO system (2) has a vector relative degree (r_1, r_2, \dots, r_m) at an operating point (x_0, u_0) if*

- (i) $\frac{\partial}{\partial u_j} h_i \circ f^p(f(x) + g(x)u) = 0, \quad \forall i = \{1, 2, \dots, m\},$
 $\forall p = \{1, 2, \dots, r_i - 2\}, \quad \forall j = \{1, 2, \dots, m\}$ for every point (x, u) in the neighbourhood of (x_0, u_0) , where u_j is the j -th element of the input u , and h_i is the i -th element of the vector function h ; and
- (ii) the decoupling matrix $D_n(x, u) \in \mathbb{R}^{m \times m}$ with elements $[D_n(x, u)]_{ij} = \frac{\partial}{\partial u_j} h_i \circ f^{r_i-1}(f(x) + g(x)u)$ has full rank at the operating point (x_0, u_0) .

Note that, from the first condition (i) of Definition 4.1, if we focus on an output dimension y_i , the relative degree r_i can be interpreted as the number of sample delays between changing any of the inputs u_j , $j = 1, \dots, m$, and changing the output y_i . Given that both (i) and (ii) of Definition 4.1 are satisfied, the relative degree r_i associates the value of an output y_i at time step $k + r_i$ with a non-zero input u applied at time step k . The decoupling matrix $D_n(x, u)$ in the second condition (ii) of Definition 4.1 is the collection of the Jacobian of $y_i(k + r_i)$ with respect to the input u ; the non-singularity condition requires that the outputs $y(k + r) = [y_1(k + r_1) \ \dots \ y_m(k + r_m)]^T$ are influenced by the input $u(k)$ in non-repeated (linearly independent) ways.

Remark 4.1. Vector Relative Degree for Linear Systems. As a special case of Definition 4.1, the linear MIMO system (1) has a vector relative degree (r_1, r_2, \dots, r_m) if

- (i) $C_i A^p B_j = 0$, $\forall i = \{1, 2, \dots, m\}$, $\forall p = \{1, 2, \dots, r_i - 2\}$, $\forall j = \{1, 2, \dots, m\}$, where C_i is the i -th row of the matrix C and B_j is the j -th column of the matrix B , and
- (ii) the decoupling matrix $D_l \in \mathbb{R}^{m \times m}$ with elements $[D_l]_{ij} = C_i A^{r_i - 1} B_j$ has full rank.

Note that, from Definition 4.1, for MIMO systems with a well-defined vector relative degree, one may relate the future output $y(k + r)$ to the current state $x(k)$ and input $u(k)$. Based on this concept, in the following subsections we provide a formal formulation of the DNN-based approach proposed in (Li et al. 2017), develop theoretical insights for systematically designing the DNN module, and provide comments on its practical implementation.

4.2 Underlying Function Modeled by the DNN Module

In this subsection we show that, given the system representations in Eqn. (1) and Eqn. (2), an identity mapping from the desired output y_d to the actual output y is achieved if the DNN module learns the output equation of the inverse dynamics of the baseline system. Due to this association with inverse dynamics, a necessary condition for the proposed approach to be effective is that the baseline system has stable inverse dynamics. For simplicity, we will start our discussion with the linear system (1) and then extend the results to the nonlinear system (2). Note that although we start our discussion with known system models, we will later demonstrate that implementing the proposed DNN-enhancement approach requires only minimal knowledge about the baseline system (e.g., the order and relative degree). This required knowledge can typically be determined from simple dynamic models or quick step response experiments.

By applying the definition of the vector relative degree in Remark 4.1 to the linear system (1), we can relate the input u and the output y of the baseline system by

$$y_i(k + r_i) = C_i A^{r_i} x(k) + C_i A^{r_i - 1} B u(k), \quad (4)$$

or in an augmented form,

$$y(k + r) = C_l x(k) + D_l u(k), \quad (5)$$

where $y(k + r) = [y_1(k + r_1) \ \dots \ y_m(k + r_m)]^T$, $C_l = [(C_1 A^{r_1})^T \ \dots \ (C_m A^{r_m})^T]^T$, and D_l is the decoupling matrix of system (1).

Let $y_d(k + r) = [y_{1,d}(k + r_1) \ \dots \ y_{m,d}(k + r_m)]^T$ be the desired output corresponding to $y(k + r)$. Since the decoupling matrix D_l has full rank by condition (ii) of the vector relative degree definition in Remark 4.1, it can be shown that if we choose the following control law

$$u(k) = D_l^{-1} (-C_l x(k) + y_d(k + r)), \quad (6)$$

then $y(k + r) = y_d(k + r)$, or exact tracking, is achieved. Thus, for the proposed DNN-enhancement control architecture in Fig. 1 and system (1), the DNN module should be trained to approximate Eqn. (6) to establish an identity mapping between y_d and y . If we consider y_d as the input and u as the output, Eqn. (6) is in fact the output equation of the inverse dynamics of system (1).

Note that the first condition (i) in Remark 4.1 implies that the relative degree r_i associated with the output dimension i is the smallest integer such that $C_i A^{r_i - 1} B_j \neq 0$ for any input dimension j . As briefly noted in Sec. 4.1, the relative degree r_i is the number of sample delays between applying an input u to the system and first seeing its effect in the particular output y_i . This inherent delay from input to output is a well-known fact for discrete-time linear systems. By training the DNN module to approximate Eqn. (6), the inherent delay of the system is compensated by the preview of the future desired output $y_d(k + r)$. In practice, at a particular time k , a preview of r steps of the desired trajectory (where $r \leq n$) is not challenging to satisfy with online or offline trajectory generation algorithms; the non-causality in Eqn. (6) is thus not an issue in practical applications.

We next generalize the previous discussion to nonlinear systems. By assuming the system (2) has a well-defined vector relative degree, and applying Definition 4.1, we can relate the input u and output y of the nonlinear MIMO system (2) by

$$y_i(k + r_i) = h_i \circ f^{r_i - 1} (f(x(k)) + g(x(k))u(k)), \quad (7)$$

or in an augmented form

$$y(k + r) = h \circ f^{r - 1} (f(x(k)) + g(x(k))u(k)), \quad (8)$$

where $h \circ f^{r - 1}$ is a vector of composition functions with the i -th element being $h_i \circ f^{r_i - 1}$. As discussed in (Sun and Wang 2001; Jang et al. 1994), by assuming $y(k + r)$ is affine in the input $u(k)$, the decoupling matrix $D_n(x, u)$ is independent of u and Eqn. (8) becomes

$$y(k + r) = h \circ f^r (x(k)) + D_n(x(k))u(k), \quad (9)$$

where $h \circ f^r$ is a composite function with the i -th element being $h_i \circ f^{r_i}$. This special case holds for nonlinear mechanical systems such as robot manipulators (Jang et al. 1994). Since the decoupling matrix D_n has full rank by the second condition (ii) in Definition 4.1, exact tracking (i.e., $y(k + r) = y_d(k + r)$) can be achieved by choosing the control law

$$u(k) = [D_n(x(k))]^{-1} (-h \circ f^r (x(k)) + y_d(k + r)), \quad (10)$$

for the affine case in Eqn. (9), and it is reasonable to assume that

$$u(k) = F(x(k), y_d(k+r)) \quad (11)$$

for the general case in Eqn. (8), where $F : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^m$ is a vector of nonlinear functions.

Based on the above results, we now present our insight on the underlying function modeled by the DNN module, and describe the conditions that are necessary for the learning-based approach to be effective:

Insight 4.1. Underlying Function and Necessary Conditions. *Consider the DNN-enhancement control architecture in Fig. 1. In order to establish an identity mapping between the desired output y_d and the actual output y , the DNN module should approximate the output equation of the baseline system's inverse dynamics. Due to the association with inverse dynamics, two necessary conditions for the learning approach to be effective are: (i) the baseline system has a well-defined (vector) relative degree; and (ii) the baseline system has stable zero dynamics.*

By inspecting the control laws in Eqn. (6) and Eqn. (11), it can be seen that the ideal control law that leads to exact tracking is dependent on the current $x(k)$ and the future desired output $y_d(k+r)$ for either the linear or nonlinear case, where r is the vector relative degree. In practice, when training the DNN module to approximate the control law for achieving exact tracking, we do not require a detailed dynamic model of the system. Instead, we need only identify the vector relative degree r of the baseline system. Experimentally, for the linear system (1) and the special case of the nonlinear system (2) where $y(k+r)$ is affine in $u(k)$, one can identify the vector relative degree of the baseline system through m step response experiments detailed as follows. In each of the m experiments, the system is initialized at an equilibrium point, and one element of the input, u_j , is activated. Without loss of generality, we assume the equilibrium is the origin. After the m experiments, one may determine the minimum number of time delays between the output y_i and the inputs u_j for all j ; the minimum number of time delays for the output dimension y_i is the estimated relative degree r_i associated with the particular output dimension. After estimating the relative degree for each output dimension, it remains to check the non-singularity condition (ii) in Definition 4.1. From the m experiments, one may construct a matrix \tilde{D} , where the j -th column of \tilde{D} is $[y_1(r_1) \cdots y_m(r_m)]^T$ from the j -th experiment. By inspecting Eqn. (5) and Eqn. (9), it can be shown that the non-singularity condition (ii) in Definition 4.1 can be examined from the rank of \tilde{D} .

The stability of the zero dynamics of the linear system (1) is equivalent to the stability of the system's inverse dynamics, and is characterized by the zeros of the system transfer function. In practice, for linear systems, we may infer the stability of zero dynamics from characteristics of the system's step responses such as undershoot and zero crossings (Hoagg and Bernstein 2007). The zero dynamics of the nonlinear system (2) is the system's invariant dynamics when the input $u(k)$ is chosen such that $y(k) = 0$ for all k . For nonlinear systems, achieving stable zero dynamics is a necessary but not sufficient condition for achieving stable inverse dynamics (Sussmann 1990). Hence, a necessary

condition for applying the proposed DNN-learning approach to either the linear system (1) or the nonlinear system (2) is that the baseline system has stable zero dynamics.

4.3 DNN Input Selection

In this subsection, we identify the necessary and sufficient inputs of the DNN module to compute the reference $u(k)$ of the baseline system (1) and (2) to achieve exact tracking. By designating the output of the DNN module as $\mathcal{O} = \{u(k)\}$, we can determine the appropriate DNN input \mathcal{I} for either the linear or the nonlinear case based on the following insight.

Insight 4.2. DNN Input Selection. *In order to establish an identity mapping from y_d to y , the necessary and sufficient input of the DNN add-on module is $\mathcal{I} = \{x(k), y_d(k+r)\}$, where $y_d(k+r) = [y_{1,d}(k+r_1) \cdots y_{m,d}(k+r_m)]^T$ and $r = (r_1, \dots, r_m)$ is the vector relative degree of the system.*

Insight 4.2 directly follows from the fact that the DNN should approximate the baseline system inverse to achieve unity mapping between y_d and y and from Eqn. (6) and Eqn. (11) of the system inverse.

The implementation of Insight 4.2 requires knowledge or estimation of the full state of the system x . In many robotics applications, linearization techniques are used for the baseline system controller designs, and this often leads to decoupled linear dynamics. Some examples include ground vehicles in which the dynamics in the 2-dimensional position space can be converted to decoupled integrators with the point-ahead linearization technique (Giesbrecht et al. 2009), and fully-actuated manipulators in which the dynamics in the joint space can be turned into decoupled double integrators with feedback linearization (Helwa and Schoellig 2016). In cases where the full state of the system is not available, but where the closed-loop dynamics can be approximated as a decoupled MIMO linear system, we can derive an alternative DNN input selection.

In deriving the alternative input selection, we first equivalently represent system (1) by $Y(z) = H(z)U(z)$, where

$$H(z) = C(zI - A)^{-1}B, \quad (12)$$

and $U(z)$ and $Y(z)$ are the z -transform of the input and output of the baseline system, respectively. To show the main idea, we first consider the special case of a SISO linear system (i.e., $m = 1$). Without loss of generality, we assume that the SISO system is represented by a transfer function of the following form:

$$H(z) = \frac{Y(z)}{U(z)} = \frac{\beta_{n-r}z^{n-r} + \beta_{n-r-1}z^{n-r-1} + \cdots + \beta_0}{z^n + \alpha_{n-1}z^{n-1} + \cdots + \alpha_0}, \quad (13)$$

where α_i and β_i are scalar constants, and r and n are the relative degree and degree of the system respectively. By calculating the inverse system of Eqn. (13) and applying inverse z -transformation, it can be shown that the reference

$u(k)$ for achieving exact tracking is

$$\begin{aligned} u(k) = & \frac{1}{\beta_{n-r}} y_d(k+r) + \frac{\alpha_{n-1}}{\beta_{n-r}} y_d(k+r-1) + \dots \\ & + \frac{\alpha_0}{\beta_{n-r}} y_d(k-n+r) - \frac{\beta_{n-r-1}}{\beta_{n-r}} u(k-1) \\ & - \frac{\beta_{n-r-2}}{\beta_{n-r}} u(k-2) - \dots - \frac{\beta_0}{\beta_{n-r}} u(k-n+r). \end{aligned} \quad (14)$$

Based on Eqn. (14), we can alternatively select DNN input for a SISO linear baseline system to be $\mathcal{I} = \{y_d(k-n+r : k+r), u(k-n+r : k-1)\}$, where the column ‘:’ abbreviates consecutive discrete-time indexes.

The transfer matrix $H(z)$ of a decoupled MIMO linear system (1) is a diagonal matrix; the dynamics between each input-output pair (u_i, y_i) can be considered separately, where $i \in \{1, \dots, m\}$. As outlined in Sec. (4.2), one can execute m experiments to identify the relative degree r_i for each output y_i . Similar to the SISO scenario discussed above, in the case of the decoupled MIMO linear system, we can consider each reference dimension separately and train m networks with the input of each network being $\mathcal{I}_i = \{y_{d,i}(k-n+r_i : k+r_i), u_i(k-n+r_i : k-1)\}$ and output being $\mathcal{O} = \{u_i(k)\}$, where r_i is the relative degree corresponding to the i -th output dimension, and $y_{d,i}$ denotes the i -th desired output dimension.

Insight 4.3. Alternative Input Selection for Decoupled MIMO Linear Systems. *Based on the transfer function formulation, we can derive an alternative, sufficient input selection of the DNN module for a decoupled MIMO linear systems. For this case, we propose using a DNN module with m independent networks – one for each of the baseline system reference dimensions. The input to the i -th network in the DNN module is $\mathcal{I}_i = \{y_{d,i}(k-n+r_i : k+r_i), u_i(k-n+r_i : k-1)\}$, where r_i is the relative degree corresponding to the output dimension y_i .*

In comparison with the input selection based on the state space representation (Insight 4.2), the implementation of the alternative input for the linear systems does not require the estimation of the full state $x(k)$ of the system and instead only requires the identification of the order of the system n , which (for robots such as multi-link manipulators) can be determined from the laws of physics. Note, however, that this transfer function approach is derived for decoupled linear systems; the state space approach is applicable to more general cases. When the state of the system is available, applying the state space approach has additional advantages. One advantage of the state space approach is the current state feedback to the DNN module. This additional feedback from the baseline system can help compensate for the initial errors and disturbances along the trajectory. Moreover, the input selection based on the state space approach typically leads to a DNN with a lower input dimension than the transfer function approach. As an example, for a SISO linear system, the dimension of the DNN inputs derived from the transfer function and the state space approaches are $(2n - r + 1)$ and $(n + 1)$, respectively. This reduced DNN input dimension implies that the amount of data required to cover the operational space is potentially less, and thus the DNN

training can be made more efficient by using the state space approach.

4.4 Stability

In this subsection, we restrict our discussion to minimum phase systems (i.e., systems with stable inverse dynamics), and prove the stability of the overall DNN-enhancement control system in the presence of DNN modeling errors:

$$\|u(k) - \hat{u}(k)\| \neq 0, \quad (15)$$

where $u(k)$ corresponds to the exact inverse in Eqn. (11) (Eqn. (6) for the linear system case) and $\hat{u}(k)$ corresponds to the reference outputted by the DNN module trained based on the system input-output data. Note that, in the ideal case, where the DNN models the inverse dynamics exactly, the response from the desired output to the actual output is the identity mapping, and the overall system is input-to-state stable. However, in the presence of modeling errors, due to the state feedback connection to the DNN module (see Fig. 1), the stability of the overall system needs to be assessed. In this subsection, we show that under Assumptions (A1) and (A3), the DNN-enhanced system with the proposed input selection as in Insight 4.2 is input-to-state stable if the regression error of the DNN module is sufficiently small.

By assumption (A3), the DNN module has a feedforward architecture, and the activation functions are globally Lipschitz; since the DNN is a composite of linear combinations of Lipschitz functions, the output of the DNN module, \hat{u} , is globally Lipschitz in its inputs, x and y_d . Moreover, the reference corresponding to the exact inverse $u(k)$ of the system is bounded by assumption. As a result of the global Lipschitz condition of the DNN module and the boundedness of u , an upper bound on the modeling error can be expressed as follows:

$$\|u - \hat{u}\|_\infty \leq L_4 \|x\|_\infty + L_5 \|y_d\|_\infty + L_6, \quad (16)$$

where L_4, L_5 , and L_6 are positive, constant scalars.

Lemma 4.1. Stability. *Consider the DNN-enhancement control architecture (Fig. 1) and the case where the baseline system is minimum phase. Under assumptions (A1) and (A3), the overall DNN-enhanced system is input-to-state stable if $L_1 L_4 < 1$, where L_1 and L_4 are constant scalars defined in Eqn. (3) and Eqn. (16), respectively.*

Proof. By assumption (A1), the baseline system is input-to-state stable, and with \hat{u} as the system input, the state of the system is bounded by

$$\|x\|_\infty \leq L_1 \|\hat{u}\|_\infty + L_2 \|x_0\| + L_3. \quad (17)$$

By combining the bound on the regression error in Eqn. (16) and the bound on state in Eqn. (17), the following is obtained:

$$\|x\|_\infty \leq L_1 \|u - \hat{u}\|_\infty + L_1 \|u\|_\infty + L_2 \|x_0\| + L_3 \quad (18)$$

$$\leq L_1 L_4 \|x\|_\infty + L_1 L_5 \|y_d\|_\infty + L_7, \quad (19)$$

where $L_7 = L_1 L_6 + L_1 \|u\|_\infty + L_2 \|x_0\| + L_3$. Based on Eqn. (19), if

$$L_4 < \frac{1}{L_1}, \quad (20)$$

is satisfied, then the state of the system is bounded by

$$\|x\|_\infty \leq \frac{L_1 L_5 \|y_d\|_\infty + L_7}{1 - L_1 L_4}, \quad (21)$$

which is bounded by a constant for bounded input y_d . Thus, if $L_4 < \frac{1}{L_1}$, then the DNN-enhanced system is input-to-state stable. \square

Note that, by examining Eqn. (17) and Eqn. (16), L_1 is a constant characterizing the maximum possible gain of the baseline system, while L_4 is a constant associated with the regression error of the DNN model. Hence, the condition in Eqn. (20) implies that if the regression error of the DNN module is sufficiently small, then the overall DNN-enhancement control architecture is input-to-state stable. One can notice the similarity between condition (20) and the well-known small gain theorem in robust control (Francis and Khargonekar 1995).

Also note that the globally Lipschitz activation function assumption in Assumption (A3) is sufficient to show (16) but not necessary; condition (16), and hence the proof of the lemma, may be satisfied in other scenarios.

4.5 Difference Learning Scheme for Improving the Training Efficiency

In this subsection, we derive a condition that allows us to further improve the data-efficiency of the proposed DNN-enhancement approach. This discussion is motivated by the DNN design in (Li et al. 2017), where the position terms in the DNN input and output are taken relative to the current desired and actual positions in order to simplify the training process. The basic idea of this difference learning scheme is that with the relative positions (instead of the absolute positions), the function modeled by the DNN becomes invariant under spatial translations, which reduces the amount of data needed to cover the operation space. Based on the theoretical formulations presented in the previous subsections, we derive in this section a necessary condition for the effectiveness of the difference learning scheme. This necessary condition will be further illustrated with quadrotor experiments in Sec. 6.

In order to motivate this insight on the difference learning scheme, we first focus our discussion on a SISO linear system represented by the transfer function representation in Eqn. (13). Recall that, for system (13), the control law for achieving exact tracking is in Eqn. (14), and the corresponding DNN input-output selection for learning the system inverse is $\mathcal{I} = \{y_d(k-n+r : k+r), u(k-n+r : k-1)\}$ and $\mathcal{O} = \{u(k)\}$, where ‘:’ is used to abbreviate consecutive time indexes. With the difference learning scheme, we aim to train a DNN that depends only on a set of relative terms, which we denote by $\Delta_{y_d}(k+p) := y_d(k+p) - y_d(k)$, where k is the current time index and $p \in \{-n+r, \dots, r\}$, and $\Delta_u(k+p) := u(k+p) - y_d(k)$, where $p \in \{-n+r, \dots, 0\}$.

Lemma 4.2. Difference Learning for SISO Linear Systems. Consider a SISO linear baseline system (13) and the DNN-enhancement control architecture in Fig. 1. A difference learning scheme can be applied to improve the data efficiency of the DNN module if and only if the baseline system has a unity DC gain.

Proof. Starting from the control law in Eqn. (14), it can be shown that by subtracting $y_d(k)$ on both sides of the equation, and adding and subtracting $\frac{1}{\beta_{n-r}} y_d(k)$, $\frac{1}{\beta_{n-r}} \sum_{i=0}^{n-r-1} \beta_i y_d(k)$ and $\frac{1}{\beta_{n-r}} \sum_{i=0}^{n-1} \alpha_i y_d(k)$ on the right-hand side, Eqn. (14) can be written as

$$\begin{aligned} \Delta_u(k) &= \frac{1}{\beta_{n-r}} \Delta_{y_d}(k+r) + \frac{\alpha_{n-1}}{\beta_{n-r}} \Delta_{y_d}(k+r-1) + \dots \\ &+ \frac{\alpha_0}{\beta_{n-r}} \Delta_{y_d}(k-n+r) - \frac{\beta_{n-r-1}}{\beta_{n-r}} \Delta_u(k-1) \\ &- \frac{\beta_{n-r-2}}{\beta_{n-r}} \Delta_u(k-2) - \dots - \frac{\beta_0}{\beta_{n-r}} \Delta_u(k-n+r) \\ &+ \frac{1}{\beta_{n-r}} \underbrace{\left(1 - \sum_{i=0}^{n-r} \beta_i + \sum_{i=0}^{n-1} \alpha_i\right)}_{\triangleq s(y_d(k))} y_d(k). \end{aligned} \quad (22)$$

In the above expression, the only non-relative time-dependent term is the last term $s(y_d(k)) = \frac{1}{\beta_{n-r}} \left(1 - \sum_{i=0}^{n-r} \beta_i + \sum_{i=0}^{n-1} \alpha_i\right) y_d(k)$ on the right-hand side. Thus, one may express the control law for achieving exact tracking in terms of the relative terms Δ_{y_d} and Δ_u (and hence apply the difference learning scheme) if and only if $s(y_d(k)) = 0$. For arbitrary $y_d(k)$, the condition $s(y_d(k)) = 0$ is equivalent to

$$\frac{\sum_{i=0}^{n-r} \beta_i}{1 + \sum_{i=0}^{n-1} \alpha_i} = 1. \quad (23)$$

For system (13), the condition in Eqn. (23) is equivalent to the condition that system (13) has a unity DC gain, i.e., it achieves zero steady state errors for step inputs. \square

We prove the same result for the MIMO state space formulation for the special case of a position/velocity-like system.

Definition 4.2. Position/Velocity-Like System. The position/velocity-like system (1) has the following properties: (i) the output of the system y is the first m elements of the state vector (i.e., x_1, \dots, x_m); and (ii) for step inputs, the remaining elements of the state vector (i.e., x_{m+1}, \dots, x_n) are zero at the steady state.

Examples of position/velocity-like systems include but is not limited to mechanical systems with a position-velocity state space (e.g., industrial manipulators). Similar to the SISO transfer function scenario, we identify a necessary condition that allows us to express the control law in Eqn. (6) in relative terms $\Delta_x(k) = x(k) - [y_d(k)^T \ 0 \ \dots \ 0]^T$, $\Delta_{y_d}(k+r) = y_d(k+r) - y_d(k)$, and $\Delta_u(k) = u(k) - y_d(k)$. In particular, we prove the following lemma by contradiction.

Lemma 4.3. Difference Learning for MIMO Linear Systems. Consider a position/velocity-like MIMO system and the DNN-enhancement control architecture in Fig. 1. A DNN design based on the state space approach (Insight 4.2) and the difference learning scheme is able to achieve exact tracking only if the baseline system has zero steady state errors for step inputs.

Proof. Suppose by the way of contradiction that the DNN-based approach achieves exact tracking for arbitrary feasible

trajectories and the baseline system does not achieve zero steady state error for an arbitrary step input $u(k) = a$, where $a \in \mathbb{R}^m$ is a constant vector. Hence, we have

$$y_{ss} = Ku_{ss} = Ka, \quad (24)$$

where $K \in \mathbb{R}^{m \times m}$ is a constant non-zero matrix characterizing the DC gains of the system, and $K \neq I_m$ by assumption, where I_m denotes the identity matrix. Note that, when K is a zero matrix, the system has zero DC gain; it can be easily shown that the mapping $\{\Delta_x(k), \Delta_{y_d}(k+r)\} \rightarrow \{\Delta_u\}$ is one-to-many and cannot be represented by the DNN module (Jordan and Rumelhart 1992). Next, for the case where K is non-zero, by assumption, the DNN module is able to achieve exact tracking $y_{ss} = y_d(k)$ for an arbitrary step input $y_d(k) = b$, where $b \in \mathbb{R}^m$ is a constant vector. With the difference learning scheme, the inputs to the DNN module are Δ_x and Δ_{y_d} and the output is Δ_u . When exact tracking is achieved, $\Delta_x = 0$ and $\Delta_{y_d} = 0$, while $\Delta_u = c$, where $c \in \mathbb{R}^m$ is a constant corresponding to the bias of the DNN model (i.e., the output of the DNN model when the inputs are zero). At the steady state, the reference of the baseline system is $u_{ss} = b + c$. From Eqn. (24), the system output at the steady state is $y_{ss} = K(b + c)$. Since exact tracking is achieved by assumption, $y_{ss} = b$ and

$$Kc = (I_m - K)b. \quad (25)$$

Since K is non-zero and $K \neq I_m$ by assumption, Eqn. (25) implies that the bias of the DNN, c , is correlated with the step input vector b . For a typical feedforward DNN, the bias c is a fixed vector determined from the training algorithm. The dependency of the bias c on the system desired output $y_d(k) = b$ leads to a contradiction. Thus, a DNN module trained with the difference learning scheme cannot achieve exact tracking for a baseline system for which the steady state error for step inputs is not zero. \square

Note that, in the above discussion, the input and output of the DNN module are taken relative to the current desired output $y_d(k)$. In practice, the input and output of the DNN module can be alternatively taken relative to the current actual output $y(k)$ to additionally compensate for initial tracking errors or disturbances.

Based on the above theoretical results for linear systems, we present the following important insight.

Insight 4.4. Necessary Condition for Applying the Difference Learning Scheme. *In order to reduce the amount of training data, a difference learning scheme can be applied to the input and output selection of the DNN module. However, as shown in the theoretical analysis for the linear system formulations, for the DNN approach with the difference learning scheme to be effective, the baseline system controller needs to be designed such that the system response has zero or sufficiently small steady-state errors for step inputs.*

The insight above is motivated from the linear system formulations. Since nonlinear systems can be approximated by a set of piecewise linear/affine systems with arbitrary accuracy (Helwa and Caines 2015), it is reasonable to expect that the necessary condition is also required for the nonlinear system (2). In Sec. 6, we verify this necessary condition for nonlinear systems with quadrotor experiments.

5 Simulation

In this section, we illustrate Insights 4.1-4.3 by considering two linear MIMO baseline closed-loop systems represented by the following state space equations:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.2 & 1 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.6 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0.5 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 0.35 & 0.35 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} x(k), \end{aligned} \quad (26)$$

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.2 & 1 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.6 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0.5 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} -0.35 & 0.35 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} x(k). \end{aligned} \quad (27)$$

Note that both system (26) and system (27) are third-order systems with 3 stable poles at $\{0.2, 0.5, 0.6\}$ and have a vector relative degree of $(1, 1)$. The only difference between these systems is the location of zero*. System (26) has a stable (minimum phase) zero at -0.8 , while system (27) has an unstable (nonminimum phase) zero at 1.2 .

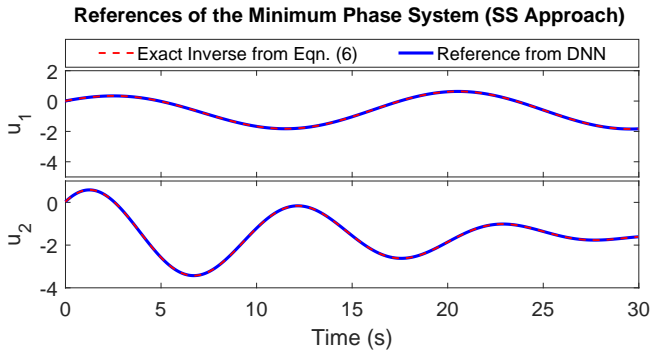
Upon introducing the DNN architecture and training in Sec. 5.1, we first follow the state space approach to select the input of the DNN module and show the necessity of stability of the baseline system zero dynamics (Insight 4.1 and Insight 4.2) in Sec. 5.2. After verifying the first two insights, we illustrate in Sec. 5.3 the efficacy of the alternative DNN input selection derived from the transfer function formulation (Insight 4.3). Note that, for this simulation study and with known system matrices (A, B, C) , we can compute Eqn. (6) and use it as the ground truth to assess the proposed DNN approach.

5.1 DNN Architecture and Training

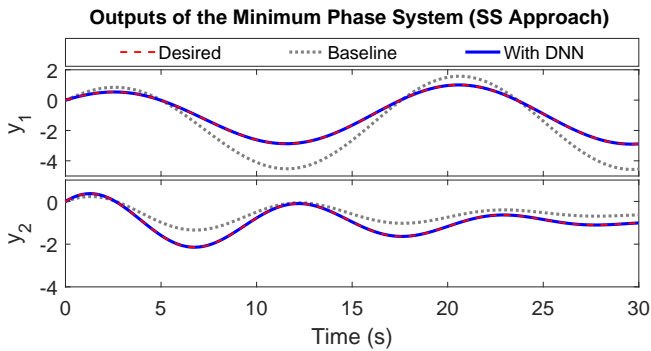
For comparison purposes, the DNN architecture and training trajectories are identical for all simulation cases presented in this section. Matlab's Neural Network Toolbox is used for implementing the DNN modules. The DNNs are a fully-connected feedforward networks with two hidden layers; each hidden layer consists of 20 hyperbolic tangent activation units. The training trajectories are 25 sinusoidal trajectories with different combinations of amplitudes and frequencies; the amplitudes range between 1 and 5, and the frequencies range between 0.024 Hz and 1.25 Hz.

The responses of the baseline system $(x(k), y(k), u(k))$ are recorded at 70 Hz for constructing the training dataset, which consists of labeled input-output pairs $(\mathcal{I}, \mathcal{O})$. In the first set of simulations, the state space approach (Insight 4.2) is examined. For both system (26) and system (27), the input and output of the DNN module are selected as $\mathcal{I}_{ss} =$

*Both system (26) and system (27) are controllable and observable and are thus minimal state space realizations. The zeros of the MIMO systems are frequencies at which the system matrix of the MIMO systems or the equivalent transfer matrices $H(z)$ of the systems drop rank (see (Dahleh et al. 2004) for more details). The locations of zeros (and poles) of the systems can be verified with the Matlab command `pzmap`.



(a) References u of the minimum phase system (26) with the state space approach (Insight 4.2). The RMS modeling error of the DNN module is approximately 7.8×10^{-5} .

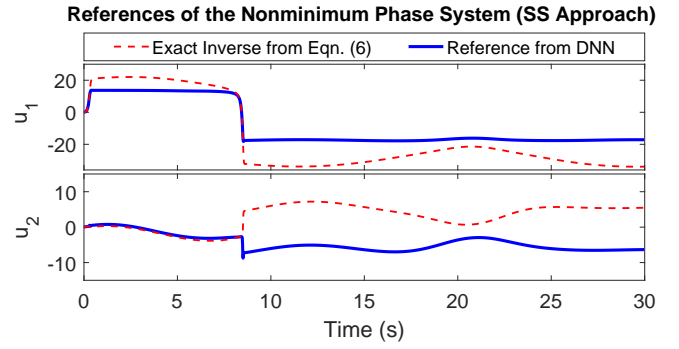


(b) Outputs y of the minimum phase system (26). The RMS tracking errors of the baseline system and the DNN-enhanced system are approximately 1.0 and 2.5×10^{-5} , respectively.

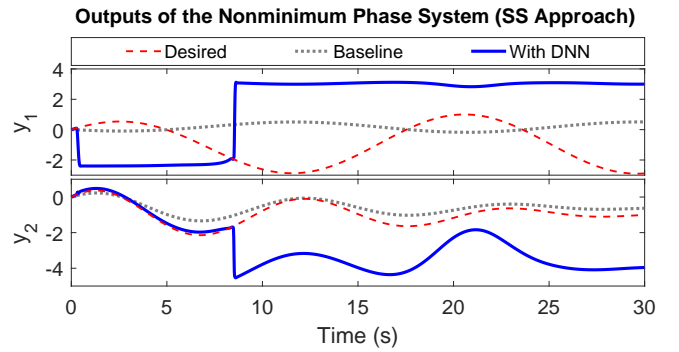
Fig. 2. The references and outputs of the minimum phase closed-loop system (26) for a desired trajectory with $y_{d,1}(t) = \sin\left(\frac{4\pi}{33}t\right) + \cos\left(\frac{4\pi}{41}t\right) - 1$ and $y_{d,2}(t) = \sin\left(\frac{4\pi}{23}t\right) + \cos\left(\frac{4\pi}{21}t\right) - 1$. From (a), the DNN module design based on Insight 4.2 is able to approximate the system's exact inverse equation (Eqn. (6)) with high accuracy; from (b), the reference computed by the DNN module is able to compensate for the errors in the baseline system response and approximately achieve exact tracking.

$\{x(k), y_{d,1}(k+1), y_{d,2}(k+1)\}$ and $\mathcal{O} = \{u(k)\}$, where $y_{d,i}$ denotes the i -th element of y_d . In the second set of simulations, we focus on the minimum phase system (26). Based on the transfer function approach (Insight 4.3), the input and output of the DNN module are $\mathcal{I}_{tf} = \{y_{d,1}(k-2:k+1), y_{d,2}(k-2:k+1), u(k-2:k-1)\}$ and $\mathcal{O} = \{u(k)\}$, where ‘:’ abbreviates consecutive discrete-time indexes. Note that, in the construction of the training dataset, the data pairs $(\mathcal{I}, \mathcal{O})$ are randomly sampled from the 25 training trajectories with balanced proportions to prevent the model from overfitting a particular frequency.

The Levenberg-Marquardt algorithm is used for training the weight and bias parameters of the DNN module. In the first set of simulations, the training objective is to minimize the mean squared error between the targets \mathcal{O} and the DNN outputs. For the second set of simulations, we additionally include an L_2 regularization term in the training objective function to help the training algorithm to phase out any unnecessary dimensions in the DNN input \mathcal{I}_{tf} ; the regularization constant is set to 0.005. In the training of each DNN module, 70% of the data is used for optimizing the model parameters and the rest is used for model validations.



(a) References u of the nonminimum phase system (27) with the state space approach (Insight 4.2). The RMS modeling error of the DNN module is approximately 14.5.



(b) Outputs y of the nonminimum phase system (27). The RMS tracking errors of the baseline system and the DNN-enhanced system are approximately 2.0 and 4.6, respectively.

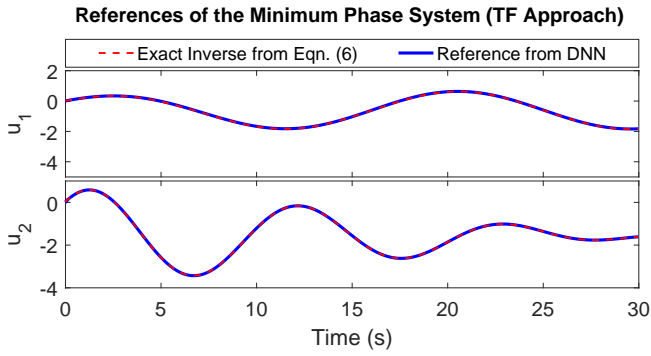
Fig. 3. The reference and outputs of the nonminimum phase closed-loop system (27) for the desired trajectory shown in Fig. 2. Due to the inherent instability of the nonminimum phase system, the reference u for achieving exact tracking is unbounded (Hoagg and Bernstein 2007). From (a), the DNN module consequently cannot effectively model the exact inverse of system (27); from (b), when the necessary condition of achieving stable zero dynamics in Insight 4.1 is violated, the DNN inverse learning approach cannot be directly applied to enhance the tracking performance of the baseline system.

The generalizability of the DNN modules is further verified by testing the tracking performance of the overall DNN-enhanced system on test trajectories that differ from the training trajectories.

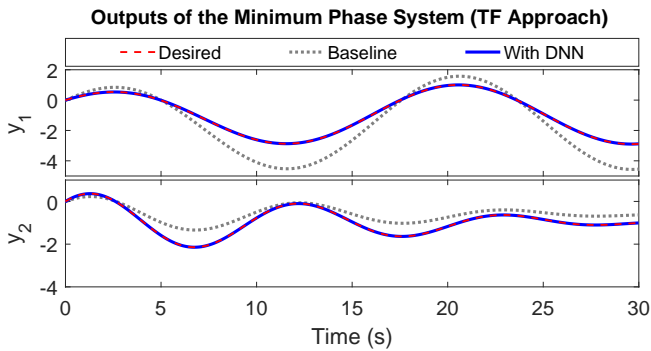
5.2 Simulation 1: Illustrations of Underlying Function and Necessary Condition

In this subsection, we illustrate Insight 4.1 and Insight 4.2 by using the state space approach and comparing the DNN-enhanced performance of the minimum phase system (26) and the nonminimum phase system (27). For this simulation illustration, the systems' performances are compared on a test trajectory that differs from those in training: $y_{d,1}(t) = \sin\left(\frac{4\pi}{33}t\right) + \cos\left(\frac{4\pi}{41}t\right) - 1$ and $y_{d,2}(t) = \sin\left(\frac{4\pi}{23}t\right) + \cos\left(\frac{4\pi}{21}t\right) - 1$.

The references and outputs of the DNN-enhanced tracking for system (26) and system (27) are shown in Fig. 2 and Fig. 3, respectively. It can be seen from Fig. 2a that, by selecting the DNN input as $\mathcal{I} = \{x(k), y_d(k+r)\}$, the DNN is able to effectively generalize the training data collected from the minimum phase system (26), and outputs references



(a) References u of the minimum phase system (26) with the transfer function approach (Insight 4.3). The RMS modeling error of the DNN module is approximately 1.2×10^{-2} .



(b) Outputs y of the minimum phase system (26). The RMS tracking errors of the baseline system and the DNN-enhanced system are approximately 1.0 and 4.2×10^{-3} , respectively.

Fig. 4. The references and outputs of the minimum phase closed-loop system (26) for the desired trajectory shown in Fig. 2. From (a), the DNN module design based on the transfer function approach (Insight 4.3) is an equivalent approximation of the exact inverse equation (Eqn. (6)); from (b), as with the state space approach (Fig. 2b), exact tracking is approximately achieved with the DNN module design based on the alternative transfer function formulation.

(blue solid line) that coincide with the reference computed based on the exact inverse in Eqn. (6) (red dashed line). With Eqn. (6) as the ground truth, the RMS modeling error of the DNN module is approximately 7.8×10^{-5} . From Fig. 2b, we see that the reference computed by the DNN module compensates for the magnitude errors in the baseline system response (grey dotted line), and leads to approximately exact tracking (blue solid line and red dashed line). On this particular test trajectory, the addition of the DNN module reduces the RMS tracking error from approximately 1.0 to approximately 2.5×10^{-5} . In this simulated setting, the performance of the proposed DNN approach is only limited by the modeling accuracies and numerical precisions. In contrast to the minimum phase case, the reference for achieving exact tracking is unbounded in the nonminimum phase system case (27) due to the inherent instabilities of the system inverse dynamics (Hoagg and Bernstein 2007). In the nonminimum phase case reflected in Fig. 3, though with the same architecture and training, the DNN module cannot effectively model the exact inverse in Eqn. (6) (Fig. 3a) and leads to worse performance as compared to the baseline system (Fig. 3b).

5.3 Simulation 2: Illustrations of the Transfer Function Approach

In the previous subsection, we showed the effectiveness of the state space approach for designing the DNN module to enhance the tracking performance of the minimum phase system (26). In this subsection, we provide a brief discussion on a DNN design based on the equivalent transfer function formulation (Insight 4.3).

Fig. 4 shows the references and outputs of the system (26) with the DNN design based on Insight 4.3. From Fig. 4a, we can see that similar to the state space approach, the DNN module design based on the transfer function approach (blue solid line) is able to approximate the reference from the exact inverse equation (Eqn. (6)) (red dashed line). For this particular test trajectory, the RMS modeling error of the DNN is approximately 1.2×10^{-2} . Consequently, as shown in Fig. 4b, the output of the DNN-enhanced system (blue solid line) also coincides with the desired trajectory (red dashed line). The RMS tracking error of the DNN-enhanced system is approximately 4.2×10^{-3} . This simulation example shows that the transfer function approach and the state space approach can be equivalently used to enhance the tracking performance of the minimum phase system (26).

6 Quadrotor Experiments

This section presents the results of quadrotor experiments designed to verify the theoretical insights derived in Sec. 4. In order to test the effectiveness the DNN module design based on the theoretical insights, we adopt the fly-as-you-draw application setup from (Li et al. 2017), where visitors are invited to draw desired trajectories on a mobile device, and the desired trajectories are tracked by a quadrotor vehicle. A demonstration video of the experiments is available at <http://tiny.cc/impromptuTracking> (Extension 1), and the hand drawings used for evaluating the proposed DNN-enhancement trajectory tracking approach are shown in Fig. 5.

In the following discussion, we first introduce the experiment setup, the control architecture, and the DNN architecture and training procedures in Sec. 6.1. In Sec. 6.2 we verify the proposed DNN input-output design and demonstrate the generalizability of the DNN module on the same 30 test trajectories. Upon verifying the proposed DNN input-output design, in Sec. 6.3 we show that the performance of the proposed approach can be pushed further by improving the representativeness of the DNN training dataset. This section is concluded with illustrations of the improved training data efficiency of the difference-learning scheme in Sec. 6.4. The training data and testing results presented in the following subsections can be found in Extension 2.

Note that, for the convenience of the discussion, we denote the desired trajectory with a subscript d , the references of the baseline system with a subscript r , and the measured states of the quadrotor with a subscript a .

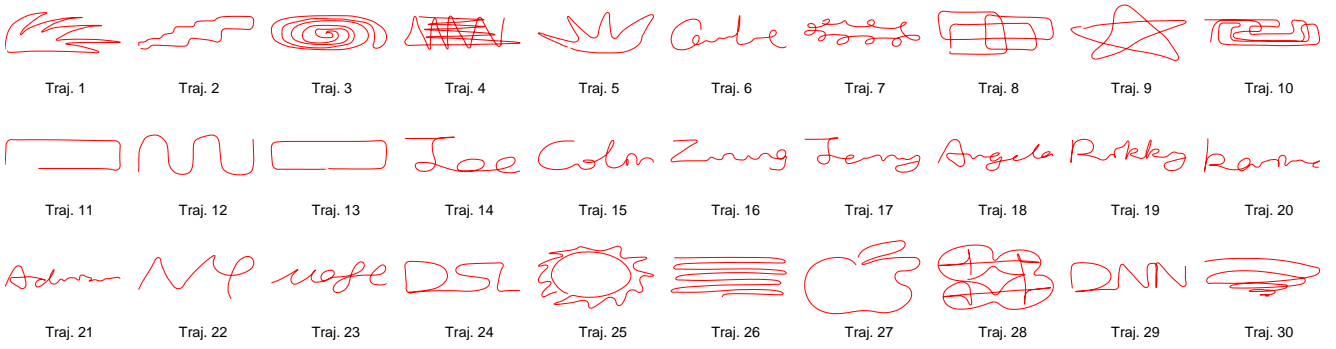


Fig. 5. Illustrations of 30 hand-drawn trajectories for testing the DNN-enhancement approach (Li et al. 2017).

6.1 Experimental Setup

6.1.1 Overview: The objective of the experiments is to design a control system such that the center of mass of a quadrotor vehicle $\mathbf{p}_a(k)$ tracks desired trajectories $\mathbf{p}_d(k)$ generated based on arbitrary hand-drawings with high accuracy from the first attempt. In the experiments, we use Parrot AR.Drone 2.0 as the testing platform and implement the control algorithm in the Robot Operating System (ROS) environment. The trajectory generation algorithm is adopted from the fly-as-you-draw application (Li et al. 2017). The estimation of the states of the quadrotor are provided by a *Vicon* motion capture system running at 200 Hz.

The 3-dimensional root-mean-square (RMS) position tracking error is used as the measure for evaluating the tracking performance of the quadrotor on a particular trajectory:

$$e_{\text{traj}} = \left(\frac{1}{N} \sum_{k=1}^N \|\mathbf{p}_d(k) - \mathbf{p}_a(k)\|^2 \right)^{\frac{1}{2}}, \quad (28)$$

where N is the number of time steps for which the trajectory is defined.

6.1.2 Control Architecture: The quadrotor vehicle has 12 states: translational positions $\mathbf{p} = (x, y, z)$, translational velocities $\mathbf{v} = (\dot{x}, \dot{y}, \dot{z})$, attitudes $\boldsymbol{\theta} = (\phi, \theta, \psi)$, and rotational velocities $\boldsymbol{\omega} = (p, q, r)$. The baseline controller of the quadrotor vehicle consists of (i) an off-board position controller that receives the reference positions and velocities (\mathbf{p}_r and \mathbf{v}_r) and outputs the desired roll angle, pitch angle, yaw rate and z -velocity commands ($\phi_{\text{cmd}}, \theta_{\text{cmd}}, r_{\text{cmd}},$ and \dot{z}_{cmd}) at 70 Hz; and (ii) an on-board attitude controller that adjusts motor thrusts based on the roll angle, pitch angle, yaw rate and z -velocity commands at 200 Hz. Of particular interest is the off-board position controller, which consists of a nonlinear transformation and PD control; from the internal model principle, it is known that this type of controller cannot be tuned to achieve perfect tracking for arbitrary desired reference frequencies (Francis and Wonham 1976). We aim to enhance the baseline position controller with our proposed DNN add-on module, which models the output of the inverse dynamics of the baseline control system. In the experiments, we introduce a DNN module design based on Insight 4.2 to adjust the position reference \mathbf{p}_r and the velocity reference \mathbf{v}_r sent to the baseline controller, and compare the tracking performance of the DNN-enhanced controller against that of the baseline

controller. Note that, as in previous work (Li et al. 2017) and for the robustness of implementation against instability, the DNN-loop in the experiments runs at 7 Hz, which is 10 times slower than the baseline controller.

6.1.3 Neural Network Architecture and Training: For comparison purposes, the DNN modules used in the experiments have the same architecture and training procedure as in (Li et al. 2017). The DNN modules are composed of fully connected FNNs with four hidden layers of 128 ReLU neurons; the python TensorFlow library is used for implementing the DNN module. The training dataset is a set of labeled input-output pairs constructed from the input-output response data of the baseline system on one or more multiple training trajectories (details of the DNN module input-output selections are discussed in Sec. 6.2). The sampling rate of the baseline system for constructing the training dataset is consistent with the update rate of the DNN module, which is selected to be 7 Hz. Of all the training data collected from the baseline system, 90% is selected for training and the remaining is used for validation. The training loss function is the squared error between the DNN output and the labeled output in the training dataset. Adam optimizer (Kingma and Ba 2014) is used for optimizing the weight parameters of the DNN. A dropout rate of 0.5 is used to improve the generalizability of the DNN to unseen inputs (Srivastava et al. 2014).

6.2 Experiment 1: DNN Input-Output Design

Through experimental trial-and-error, (Li et al. 2017) found that a DNN module with input $\mathcal{I}_1 = \{\mathbf{p}_d(k+4) - \mathbf{p}_a(k), \mathbf{p}_d(k+6) - \mathbf{p}_a(k), \mathbf{v}_a(k), \mathbf{v}_d(k+4), \mathbf{v}_d(k+6), \boldsymbol{\theta}_a(k), \boldsymbol{\theta}_d(k+4), \boldsymbol{\theta}_d(k+6), \boldsymbol{\omega}_a(k), \boldsymbol{\omega}_d(k+4), \boldsymbol{\omega}_d(k+6), \ddot{z}_a(k), \ddot{z}_d(k+4), \ddot{z}_d(k+6)\}$ and output $\mathcal{O}_1 = \{\mathbf{p}_r(k) - \mathbf{p}_d(k), \mathbf{v}_r(k) - \mathbf{v}_d(k)\}$ can effectively improve the performance of the baseline system for tracking arbitrary hand-drawn trajectories; on the 30 hand-drawn trajectories shown in Fig. 5, the average RMS error reduction achieved by the DNN module is approximately 43%. In order to verify Insight 4.2, we repeat the 30 test trajectories from (Li et al. 2017) with a DNN module design based on the proposed input-output selection and compare the improved tracking performance with that achieved in (Li et al. 2017). Note that we repeated the experiments in (Li et al. 2017) on the quadrotors used for this work for comparability.

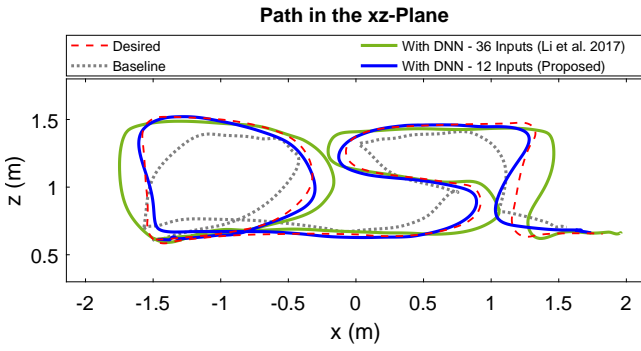


Fig. 6. A comparison of the tracking performance enhancements between the DNN module from (Li et al. 2017) and the DNN module design based on Insight 4.2 for a hand-drawn test trajectory (Traj. 24 in Fig. 5). On this test trajectory, the RMS tracking error of the baseline system is approximately 0.41 m. The RMS tracking errors of the baseline system enhanced by the DNN module from (Li et al. 2017) and the proposed DNN module design based on Insight 4.2 are 0.23 m and 0.14 m respectively, which correspond to 45% and 67% error reductions.

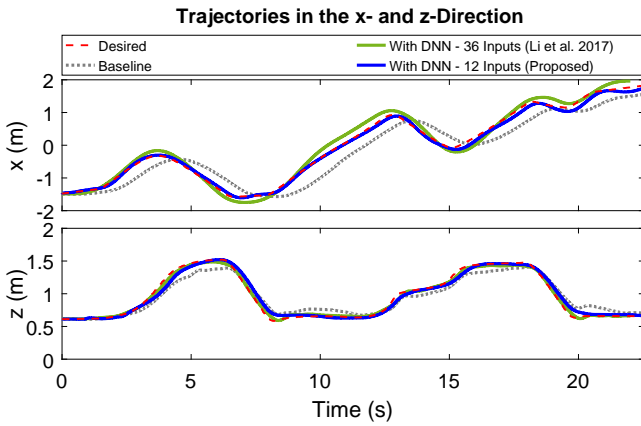


Fig. 7. A comparison of the x - and z -position trajectories for a hand-drawn test trajectory (corresponding to Fig. 6). From the plots, the DNN module from (Li et al. 2017) and the DNN module trained based on Insight 4.2 both tend to correct the delays and magnitude errors of the baseline system response. When compared to the DNN from (Li et al. 2017), the proposed DNN design based on Insight 4.2 has two thirds fewer inputs while achieving better performance enhancements.

In order to apply our insights, we first performed simple step response experiments and identified the following properties of the baseline system:

- (P1) the responses of the baseline system are approximately decoupled in the x -, y -, and z -direction;
- (P2) the relative degrees of the baseline system in the x -, y -, and z -direction are 4, 4, and 3, respectively; and
- (P3) zero steady state error for step inputs is achieved in the three directions.

Given properties (P1)-(P3), we assume decoupled dynamics in the x -, y -, and z -direction and apply Insight 4.2 with the difference learning scheme to obtain the following input-output selection of the DNN module: $\mathcal{I}_2 = \{x_d(k+4) - x_a(k), y_d(k+4) - y_a(k), z_d(k+3) - z_a(k), \dot{x}_d(k+3) - \dot{x}_a(k), \dot{y}_d(k+3) - \dot{y}_a(k), \dot{z}_d(k+2) -$

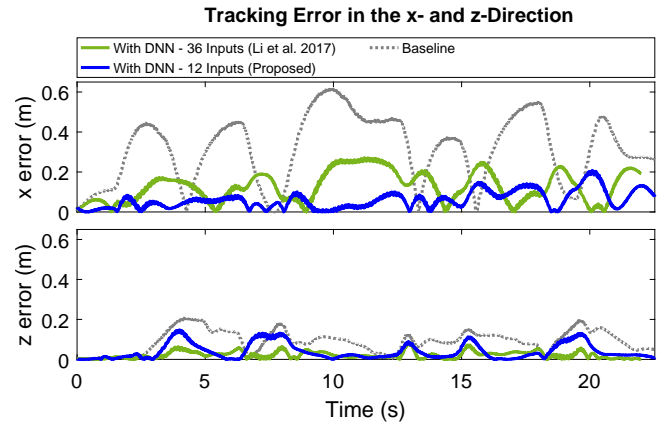


Fig. 8. The tracking errors of the x - and z -position ($|x(k) - x_d(k)|$ and $|z(k) - z_d(k)|$) corresponding to Fig. 7. The DNN module from (Li et al. 2017) and the DNN module trained based on Insight 4.2 both effectively reduce the peak tracking errors of the baseline system. The peak errors for the baseline system in the x - and z -direction are approximately 0.62 m and 0.21 m, respectively. For the DNN design from (Li et al. 2017), the peak tracking errors in the x - and z -direction are reduced to approximately 0.27 m and 0.09 m, while for the proposed DNN, the peak tracking errors in the x - and z -direction are reduced to approximately 0.21 m and 0.15 m.

$\dot{z}_a(k), \theta_a(k), \omega_a(k)\}$ and $\mathcal{O}_2 = \{\mathbf{p}_r(k) - \mathbf{p}_a(k), \mathbf{v}_r(k) - \mathbf{v}_a(k)\}$.

We note two differences between the DNN from (Li et al. 2017) and the proposed DNN design based on Insight 4.2. The first is the DNN input selection. In comparison with the DNN from (Li et al. 2017), which has 36 inputs ($\#\mathcal{I}_1 = 36$, where $\#$ denotes cardinality), the DNN design based on Insight 4.2 has only 12 inputs ($\#\mathcal{I}_2 = 12$). Based on the inverse-dynamics formulation, the input selection \mathcal{I}_2 represents the necessary and sufficient inputs that allow the DNN add-on module to achieve enhanced tracking performance. Another difference is in the application of the difference learning scheme for the two DNN designs. In particular, for the DNN design from (Li et al. 2017), the position elements in the input \mathcal{I}_1 are taken relative to the actual output values (subscripted with a), and the position elements in the output \mathcal{O}_1 are taken relative to the desired output values (subscripted with d). For the proposed DNN design based on Insight 4.2, the relative terms in the input \mathcal{I}_2 and output \mathcal{O}_2 are consistently taken with respect to the actual values (subscripted with a). Based on the theoretical discussions of Insight 4.4, we expect the consistency of the relative terms in the proposed design would further improve the capability of the DNN module in correcting for any deviations from the desired trajectories.

We first present the performance comparison between the DNN from (Li et al. 2017) and the proposed DNN on one of the test trajectories (Figs. 6, 7, and 8). We then summarize the comparison between the two DNN designs on the 30 hand-drawn trajectories (Fig. 9). Note that, for the experiments, the DNN modules are trained on a 400-second 3-dimensional sinusoidal trajectory similar to that used in (Li et al. 2017) (Extension 2a).

From Fig. 6 and Fig. 7, it can be seen that both the DNN from (Li et al. 2017) (green solid line) and the DNN design based on Insight 4.2 (blue solid line) are able to reduce the

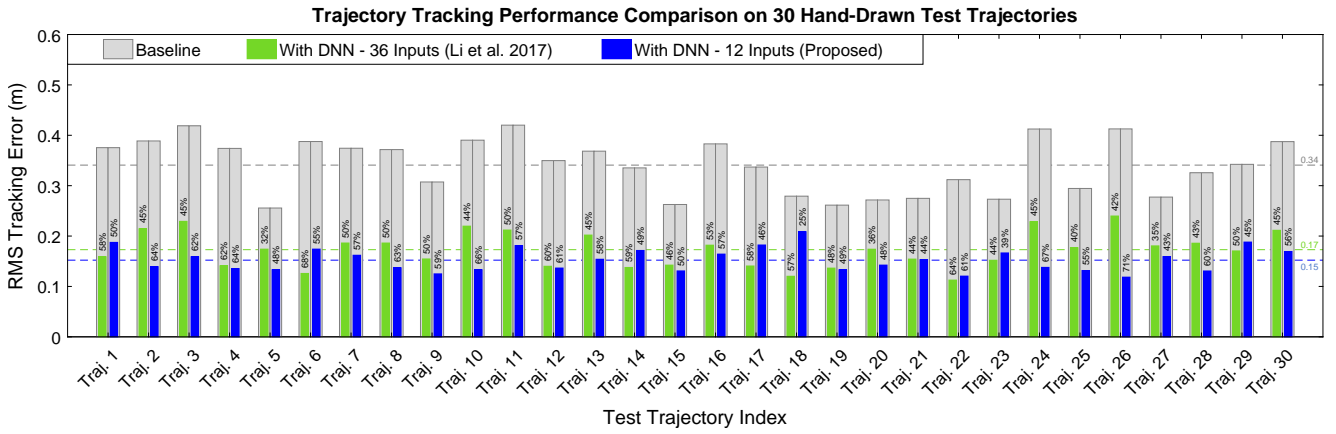


Fig. 9. Comparisons of the tracking performance enhancements between the DNN module from (Li et al. 2017) (with 36 inputs) and the proposed DNN module design based on Insight 4.2 (with 12 inputs). In the two sets of experiments, the percentage of the RMS tracking error reductions achieved by the DNN module are indicated above the corresponding bars; the mean RMS error over the 30 trajectories are indicated by the horizontal dashed lines. Note that, for comparability, we repeated the 30 impromptu tracking experiments from (Li et al. 2017) on the same quadrotor vehicles and training datasets used for testing the proposed DNN design. From the plot, despite having two thirds fewer inputs, the proposed DNN design based on Insight 4.2 yields a performance comparable to the DNN from (Li et al. 2017). On the 30 test trajectories, the average RMS error reduction is 49% for the DNN from (Li et al. 2017) and 54% for the proposed DNN design based on Insight 4.2.

time delays and magnitude errors of the baseline system tracking response (grey dotted line) and lead to quadrotor tracking paths that are closer to the desired hand-drawing (red dashed line). On this test trajectory, the RMS tracking error reduction achieved by the DNN from (Li et al. 2017) and the proposed DNN are 45% and 67%, respectively. The trajectory tracking error comparison depicted in Fig. 8 shows that the proposed DNN design based on Insight 4.2 achieves similar error reductions to the DNN design from (Li et al. 2017) while having far fewer inputs.

Fig. 9 (Extension 2b) summarizes the performance comparison between the two DNN modules on the 30 hand-drawn test trajectories studied in (Li et al. 2017) (see Fig. 5). The plot shows that the proposed DNN module design based on Insight 4.2 (blue bars) leads to similar tracking performance as the DNN module from (Li et al. 2017) (green bars). On the 30 test trajectories, the mean RMS error of the baseline system enhanced by the DNN from (Li et al. 2017) is approximately 0.17 m, and that of the baseline system enhanced by the proposed DNN is approximately 0.15 m. The corresponding average tracking error reduction achieved by the DNN from (Li et al. 2017) and that design based on Insight 4.2 are 49% and 54%, respectively.

From this set of experiments, we verify Insight 4.2 on the proposed DNN input selection. Although the input dimension is reduced by two thirds as compared with the DNN from (Li et al. 2017), the DNN module design based on the derived theoretical insight can effectively enhance the quadrotor’s baseline system performance. The comparison with the results from (Li et al. 2017) further proves the generalizability of the proposed DNN for tracking arbitrary untrained trajectories impromptu.

6.3 Experiment 2: DNN Training Dataset

In the previous set of experiments, the performance of the DNN from (Li et al. 2017) and the proposed DNN are compared on the basis of training on 400-second sinusoidal trajectories. These trajectories have gradually increasing

amplitudes but fixed frequencies in the x -, y -, and z -direction (Li et al. 2017). In this subsection, we show that the performance enhancement achievable by the proposed DNN design can be further improved with a richer training dataset. In particular, we compare two training datasets constructed from the baseline system responses to different training trajectories (Extension 2a):

- Training Dataset 1 based on a 400-second sinusoidal training trajectory, and
- Training Dataset 2 based on the 400-second sinusoidal training trajectory from Training Dataset 1 and 30 additional hand-drawn trajectories.

Note that the 30 hand-drawn trajectories in Training Dataset 2 are different from the 30 trajectories (Fig. 5) for evaluating the performance of the DNNs.

Visualizations of the training datasets in the DNN input space are shown in Fig. 10. Note that, when comparing Training Dataset 1 and Training Dataset 2, the additional data points corresponding to the drawing trajectories are reflected from the denser cores around the origins in each of the plots on the right. It is expected that the additional data would reduce the generalization errors of the DNN module when it is applied for impromptu tracking of hand-drawn trajectories.

Fig. 11 (Extension 2b) shows the performance comparison of three DNN-enhanced systems on the 30 test hand-drawn trajectories (Fig. 5). From the previous subsection, we show that, on average, the DNN with the proposed inputs (middle histogram in Fig. 11) leads to better performance as compared with the DNN from (Li et al. 2017) (top histogram in Fig. 11). When comparing the proposed DNN trained with Training Dataset 1 (middle histogram in Fig. 11) and Training Dataset 2 (bottom histogram in Fig. 11), we see that the inclusion of the additional hand-drawn trajectories in training further improves the performance of the DNN-enhanced system in tracking arbitrary hand-drawn trajectories. Overall, the proposed DNN trained with Training Dataset 2 increases the average RMS tracking error

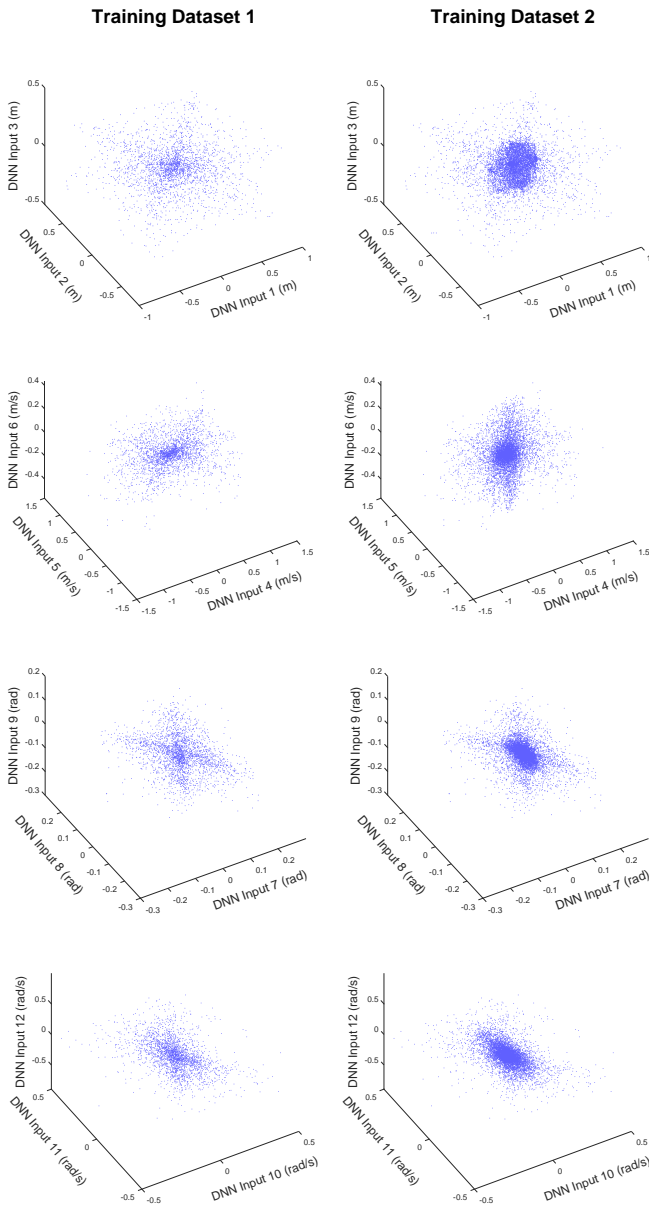


Fig. 10. Visualizations of two training datasets in the DNN input space defined by \mathcal{I}_2 . The plots on the left correspond to Training Dataset 1 (400-second sinusoidal training trajectory) and the plots on the right correspond to Training Dataset 2 (the 400-second sinusoidal trajectory with 30 additional hand-drawn trajectories). Note that, for the second training dataset, the 30 hand-drawn trajectories used for training are different from the 30 trajectories (Fig. 5) used for evaluating the tracking performance.

reduction by 8% as compared with the proposed DNN trained with Training Dataset 1.

6.4 Experiment 3: Difference Learning

In Sec. 6.4, we showed in theory that, when applying the difference learning scheme to improve the data efficiency of the DNN training, the baseline system needs to achieve zero steady state error for step inputs. In this subsection, we first illustrate the necessity of the condition by applying the difference learning scheme to DNN modules to enhance (i) the original baseline system where zero steady state error for step inputs is achieved, and (ii) a modified baseline system where the necessary condition is not achieved. In the experiment, the modified baseline system is obtained

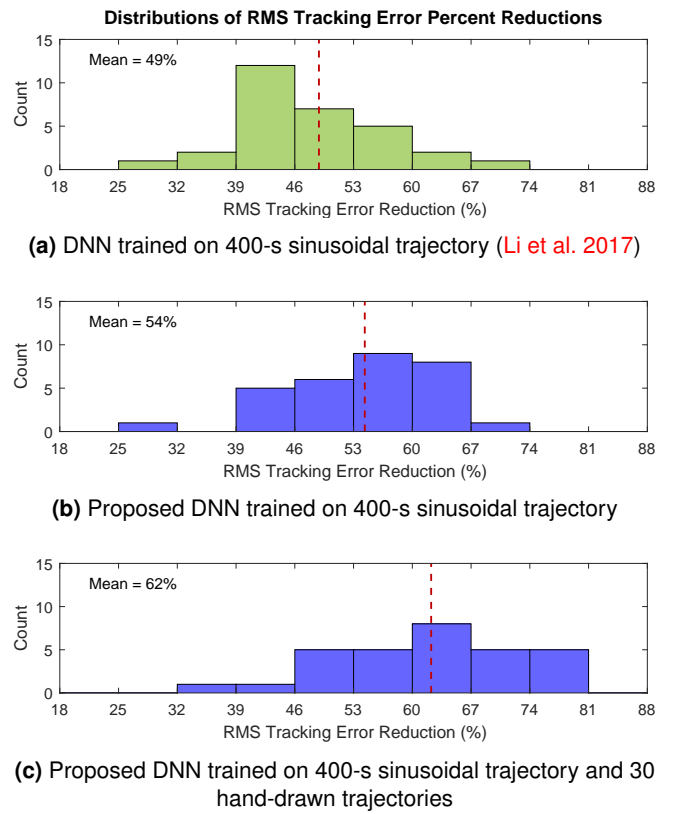


Fig. 11. A comparison of the tracking error reduction achieved by (a) the DNN used in (Li et al. 2017) and trained on a 400-second sinusoidal trajectory, (b) the proposed DNN designed based on Insight 4.2 and trained on the 400-second sinusoidal trajectory, and (c) the proposed DNN designed based on Insight 4.2 and trained on the 400-second sinusoidal trajectory and 30 additional hand-drawn trajectories. Note that, for the last case (c), the 30 additional hand-drawn trajectories used for training are different from the 30 test trajectories. The mean percent error reduction for each distribution is indicated by the vertical dashed line.

by multiplying the reference signals z_r sent to the original baseline system by a factor of 0.5. The baseline and DNN-enhanced tracking performance for the two systems are shown in Fig. 12 (Extension 2c). The plots show that for the original baseline system (bottom panel), where zero steady state error for step inputs is achieved, the DNN with the difference learning scheme is able to effectively enhance the tracking performance of the baseline system. However, as expected from Insight 4.4, for the modified baseline system (top panel), where the zero steady state error condition is not satisfied, the DNN trained with the difference learning scheme only partially compensates for the magnitude error and the bias of the modified baseline system.

Having demonstrated the necessary condition for applying the difference learning scheme, and in order to determine how much the training data efficiency can be improved, we next compare a DNN trained with the difference learning scheme and a DNN trained without it. Fig. 13 (Extension 2c) shows a comparison of the DNN modules trained with (blue) and without (red) the difference learning scheme for enhancing the tracking performance of the quadrotor baseline system where zero steady state error for step inputs is achieved. In the plot, the RMS tracking errors of the

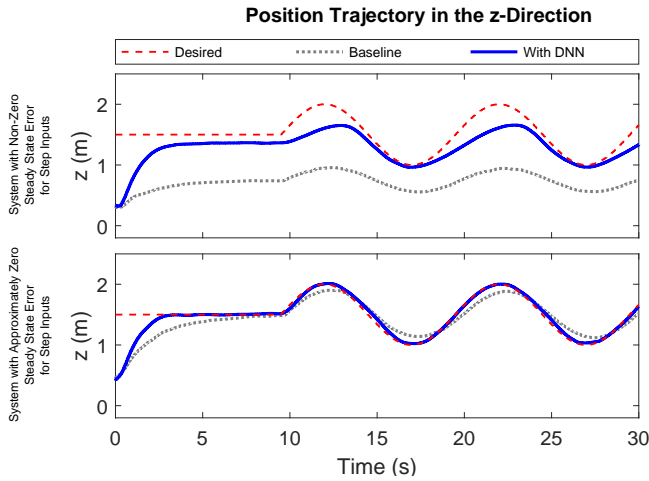


Fig. 12. A comparison of the difference learning scheme as applied on: (i) a baseline system for which zero steady state error for step inputs is not achieved (top); and (ii) a baseline system for which zero steady state error for step inputs is achieved (bottom). When the necessary condition of having a baseline system that achieves zero steady state error for step inputs is not satisfied (see Insight 4.4), the DNN trained with the difference learning scheme cannot effectively compensate for the errors of the baseline system response.

DNN-enhanced systems are compared as the amount of training data varies. Note that, in order to prevent overfitting, the training datasets are randomly sampled from a large training dataset (Training Dataset 2). Here, we use Traj. 24 (Fig. 5) as the test trajectory for evaluating the performance of the DNN-enhanced systems. The plot depicted in Fig. 13 shows that for the DNN without the difference learning scheme (red), the RMS tracking error increases quickly as the amount of training data reduces. In contrast, the performance of the DNN trained with the difference learning scheme (blue) drops more gradually as the amount of training data decreases. The DNN trained with the difference learning scheme reaches the best observed performance of the DNN without the difference learning scheme (grey dotted line), with approximately 15 times less data.

7 Discussion

In Sec. 6, we showed that the proposed DNN approach can effectively enhance the impromptu tracking performance of classical controllers. In the proposed approach, the design of the DNN module relies only on the input, output, and state data of the baseline system, as well as basic properties of the system (e.g., the vector relative degree) that can be identified from a set of simple step response experiments. Without requiring a dynamic model as a prior, the DNN approach can be used to complement black-box control systems — a possible step towards addressing the issue of uncertain or unmodeled dynamics that limit the performance of classical model-based control design approaches.

Despite its advantages, the proposed approach has limitations that require further investigation. The first relates to its applicability to systems with hybrid dynamics, where hybrid control strategies are often applied (Antsaklis 2000). In our formulation, we represent the baseline system by Eqn. (1) and Eqn. (2). For hybrid systems such as bipedal

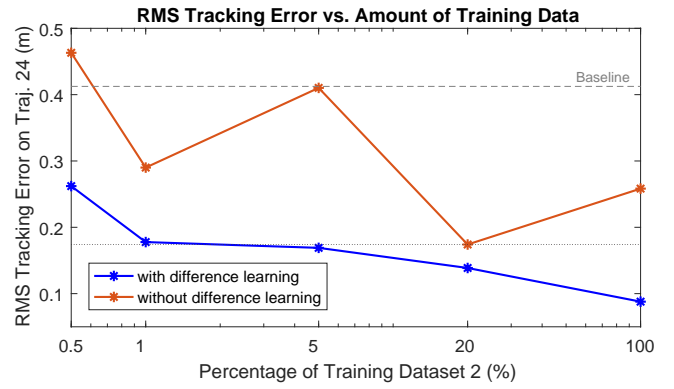


Fig. 13. A comparison of the RMS tracking error versus the amount of data for training the DNNs with (blue) and without (red) the difference learning scheme. The horizontal axis shows the proportion of randomly selected data from Training Dataset 2 described in Sec. 6.3; the vertical axis shows the RMS error on Traj. 24 with the DNN-enhanced system (see Fig. 5). The plot shows that the DNN trained with the difference learning scheme is able to reach the best observed performance of the DNN trained without the difference learning scheme (indicated by the grey dotted line) with approximately 15 times less training data. Note that the RMS tracking error corresponding to the baseline system is shown as a grey dashed line for reference.

robots or quadrotors with load suspension, however, multiple dynamic equations defined on different regions of the state space must be considered. One trivial approach is to apply the current results to each individual dynamic system and train a set of independent DNN modules to enhance the tracking performance. Accounting for transitions across the boundaries of the dynamic regions is an open question. The hierarchical structure in some typical hybrid control approaches (e.g., supervisory control) naturally encourages a hierarchical learning structure for dealing with this class of systems capturing more complex dynamics.

The second limitation is the transparency of the DNN training. In the simulations and experiments, we trained the DNN modules with datasets that sufficiently cover the operation space. Although we showed that the amount of training data can be significantly reduced by applying the difference learning scheme to the DNN module design, the sufficiency of the training dataset, and hence the performance of the DNN module, are not known prior to the tests on the physical setup. One promising direction for examining the data sufficiency is to introduce probabilistic learning to the current DNN-enhancement control framework. Many researchers are investigating approaches that provide uncertainty estimations to deep learning (e.g., (Gal and Ghahramani 2016; Depeweg et al. 2018)). As noted in (Depeweg et al. 2018), these approaches can be naturally combined with the active learning framework (MacKay 1992) to guide training data collection. The probabilistic framework and guided data collection can potentially provide indications of insufficient training and further increase training efficiency.

8 Conclusions

This paper presents theoretical and experimental studies of a DNN-based approach for enhancing the tracking

performance of black-box control systems for arbitrary feasible trajectories. We considered a MIMO, possibly nonlinear, system as our starting point. In order to achieve an identity mapping from the desired output to the actual output, we established that the DNN module in the proposed control architecture should approximate the output equation of the inverse dynamics of the baseline system. Due to the association with system inversion, the effectiveness of the proposed approach relies on two necessary conditions that the baseline system has (i) a well-defined vector relative degree and (ii) stable zero dynamics. Second, for the systems satisfying these two necessary conditions, we identified the necessary and sufficient inputs of the DNN module. Third, we verified the insights by repeating the quadrotor experiments in (Li et al. 2017). In particular, we showed that with the proposed DNN input selection, the DNN input dimension is reduced by two thirds while achieving similar or better performance on the 30 hand-drawn trajectories in (Li et al. 2017). By using a richer training dataset, we showed that the proposed DNN module reduced RMS error by approximately 62% on the average of the 30 testing hand-drawn trajectories. Fourth, using an argument similar to the small gain theorem, we proved that, for systems with stable zero dynamics, the overall DNN-enhanced control system is input-to-state stable if the DNN modeling error is sufficiently small. Fifth, we explored via both theory and experiments the effectiveness of the difference learning scheme for improving the efficiency of the training of the DNNs in the proposed approach. In particular, we derived a necessary condition for the effectiveness of the difference learning approach, and verified this condition via experiments. For the quadrotor impromptu tracking experiments, we showed that the DNN trained with the difference learning scheme is able to achieve comparable tracking performance of a DNN module trained without the difference learning scheme with approximately 15 times less data.

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Declaration of conflicting interests

The authors declare that there is no conflict of interest.

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Appendix A: Index to Multimedia Extensions

Table 1. Index to Multimedia Extensions

Extension	Media Type	Description
1	Video	Impromptu tracking experiment
2	Data	a. Training data b. Testing results on 30 trajectories c. Difference learning scheme