Abstract—Robots and automated systems are being introduced to unknown and dynamic environments where they are required to handle disturbances, unmodeled dynamics and parametric uncertainties. Robust and adaptive control strategies are required to achieve high performance in these dynamic environments. In this paper, we propose a novel adaptive model predictive controller (MPC) that combines a model predictive controller with an underlying $\mathcal{L}_1$ adaptive controller to improve trajectory tracking of a system subject to unknown and changing disturbances. The $\mathcal{L}_1$ adaptive controller forces the system to behave in a predefined way, as specified by a reference model. A higher-level MPC then uses this reference model to calculate the optimal input given a cost function while taking into account input constraints. We focus on experimental validation of the proposed approach and demonstrate its effectiveness in experiments on a quadrotor. We show that the proposed approach has a lower trajectory tracking error compared to non-predictive, adaptive approaches and a predictive, non-adaptive approach even when external wind disturbances are applied.

I. INTRODUCTION

Robots and automated systems are being introduced to unknown and dynamic environments, which requires the design of sophisticated control methods that can achieve high overall performance in these situations. Unlike traditional controllers where small changes in the conditions may significantly deteriorate the controller performance and may cause instability (see [1], [2] and [3]), controllers deployed in changing environments must be robust to model uncertainties, unknown disturbances and changing dynamics. Robotic applications in these increasingly challenging environments include autonomous driving, assistive robotics and unmanned aerial vehicle (UAV) applications. For example, in airborne package delivery, packages to be delivered can have different properties (mass, center of gravity and inertia) and may be subjected to different weather conditions. As such, it is not feasible to design a controller for each environmental or system condition.

In this paper, we present a controller that achieves high-accuracy tracking performance and is robust to unknown disturbances and changing dynamics. We consider nonlinear systems and propose a novel architecture that combines $\mathcal{L}_1$ adaptive control and model predictive control (MPC), see Fig. 1. MPC is an attractive control scheme because constraints can easily be incorporated and because of its predictive capability. It is an optimal control scheme; however, its performance depends on the accuracy of the model. To overcome this, we propose an underlying $\mathcal{L}_1$ adaptive controller which forces a potentially nonlinear system to behave in the same predefined way, as specified by a linear reference model, even in the presence of model uncertainties. As a result, a standard linear MPC is able to achieve high-accuracy trajectory tracking of the underlying nonlinear system. We validate the proposed approach in extensive experiments on a quadrotor. We show that the predictive capabilities of MPC and the robustness to disturbances of $\mathcal{L}_1$ adaptive control enable the proposed approach to achieve better trajectory tracking performance compared to non-predictive and/or non-adaptive approaches, even when the system is subject to external, unknown disturbances.

Model predictive control (MPC) yields an optimal control sequence at each sampling instant and incorporates constraints on control inputs and states. Discrete-time optimal control is concerned with choosing an optimal input sequence (as measured by some objective function), over a finite or infinite time horizon, in order to apply it to a system with a given initial state. It has been widely applied in industries where satisfaction of constraints is important [4]. The standard implementation of MPC using a nominal model of the system dynamics exhibits nominal robustness to small disturbances [5]. However, these robustness guarantees may be insufficient in practical situations where disturbances are higher.

To handle model uncertainties, adaptive MPC schemes have been introduced. These schemes update the model
online based on measurement data. In [6], an adaptive MPC method for a class of constrained linear time-invariant systems is proposed. This approach is based on a novel way to estimate the parameters of a model for system dynamics that is suitable for MPC. The algorithm is initially conservative due to large parameter uncertainty, but increases performance as the parameter estimate converges to the true value and the estimate’s uncertainty is reduced. However, the estimation depends on the excitation over the prediction horizon of a variable that is a function of the state. Similar adaptive approaches have been proposed for nonlinear systems; however, these are challenging to design and computationally expensive. An adaptive nonlinear MPC scheme for constrained nonlinear systems is presented in [7]. It uses a min-max feedback nonlinear MPC in combination with an adaptation mechanism to address the issue of robustness while the estimated value is evolving. Once again, the algorithm is initially conservative due to large parameter uncertainty, and the conservativeness reduces with improved parameter estimation. However, this approach is still computationally expensive. Finally, learning-based MPC approaches such as [8] and [9] use neural networks to learn the dynamics of the system used in the MPC implementation, which requires a significant amount of data in order to build an accurate model and is not adaptive to changes in the environment.

In light of these issues with the existing approaches, we use $L_1$ adaptive control, which is based on the model reference adaptive control (MRAC) architecture but includes a low-pass filter that decouples robustness from adaptation [10]. As a result, arbitrarily high adaptation gains can be chosen for fast adaptation. It has successfully been used to control fixed wing vehicles [11], [12], quadrotors [13], [14], a NASA Airstar flight test vehicle [15], tailless unstable aircraft [16], and quadrotor, hexacopter and octocopter vehicles [17]. $L_1$ adaptive control has been used in combination with iterative learning control (ILC) to improve trajectory tracking over iterations [14]. The ILC computes the input for the next iteration offline after the previous iteration is finished. In contrast to [14], we calculate the inputs online using MPC which enables the system to achieve high accuracy trajectory tracking on the first iteration.

The key contributions of this paper are:

- To demonstrate the effectiveness of a combined $L_1$ adaptive control and a linear MPC approach to control nonlinear systems with reduced computational cost compared to nonlinear MPC schemes.
- To remove the need for persistent excitation to achieve accurate adaptation as in existing adaptive MPC strategies [18].
- To validate the novel $L_1$ adaptive control and MPC approach through extensive experimental results.
- To demonstrate the robustness of the proposed approach to external disturbances, e.g. wind disturbance, as shown in experimental results with a quadrotor.
- To demonstrate the improved tracking performance on a quadrotor compared to two non-predictive and adaptive frameworks and a predictive and non-adaptive scheme.

The remainder of this paper is organized as follows. The problem is defined in Section II. The details of the proposed approach are presented in Section III. Experimental results on a quadrotor are presented in Section IV. Conclusions are provided in Section V.

### II. PROBLEM STATEMENT

The objective of this work is to achieve high-accuracy trajectory tracking in the presence of uncertain, and possibly changing (environmental) conditions on the first iteration. We consider a control architecture as shown in Fig. 1, where a model predictive controller is used on top of an underlying $L_1$ adaptive controller.

The $L_1$ adaptive controller forces a system, which may be nonlinear, to behave in the same predefined way, as a linear reference model. We assume that the system dynamics (‘Plant’ block in Fig. 1) are unknown, can change over time, and can be described by a single-input, single-output (SISO) system (this approach can be extended to multi-input, multi-output (MIMO) systems) identical to [10] for output feedback:

$$y_1(s) = A(s)(u_{L_1}(s) + d_{L_1}(s)),$$

where $y_1(s)$ is the Laplace transform of the translational velocity $y_1(t)$, $A(s)$ is a strictly-proper unknown transfer function that can be stabilized by a proportional-integral controller, $u_{L_1}(s)$ is the Laplace transform of the input signal, and $d_{L_1}(s)$ is the Laplace transform of the disturbance signal defined as $d_{L_1}(t) = f(t, y_1(t))$, where $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is an unknown map subject to the following assumption:

**Assumption 1** (Lipschitz continuity). There exist constants $L > 0$ and $L_0 > 0$, possibly arbitrarily large, such that the following inequalities hold uniformly for all $t$:

$$|f(t, v) - f(t, w)| \leq L|v - w|,$$

and

$$|f(t, w)| \leq L|w| + L_0.$$

The system is required to accurately track a desired trajectory $y_1^*(t)$ defined over a finite time interval. This trajectory is assumed to be feasible with respect to the true dynamics of the plant controlled by an $L_1$ adaptive controller (Fig. 1, orange dashed box). This signal is discretized as the input of computer-controlled systems is sampled and measurements are only available at fixed time intervals. We use a quadratic cost to define trajectory tracking performance, which can be minimized at every time-step to find the optimal control input.

### III. METHODOLOGY

We consider two main subsystems: the $L_1$ adaptive controller (orange dashed box in Fig. 1) and the model predictive controller. The $L_1$ adaptive controller is presented in Section III-A. Section III-B introduces the MPC. The combined $L_1$ adaptive control and MPC framework is described in Section III-C.
A. $\mathcal{L}_1$ Adaptive Control

The $\mathcal{L}_1$ adaptive controller is used to make the system behave as a pre-defined, linear system even when unknown, changing disturbances act on the system. In this work, the standard $\mathcal{L}_1$ adaptive output feedback controller for SISO systems [10] is used. For convenience and completeness, we present a brief description of the $\mathcal{L}_1$ adaptive controller.

Problem Formulation: The output feedback $\mathcal{L}_1$ adaptive controller is used to design a control input $u_{\mathcal{L}_1}(t)$ such that $y_1(t)$ tracks the bounded piecewise continuous reference input $r_1(t)$ according to the following first-order reference system:

$$M(s) = \frac{m}{s + m}, \quad m > 0.$$  \hfill (4)

Definitions and $\mathcal{L}_1$-Norm Condition: We can rewrite the system (1) in terms of the reference system (4) as follows:

$$y_1(s) = M(s)(u_{\mathcal{L}_1}(s) + \sigma(s)), \quad \text{where uncertainties in } A(s) \text{ and } d_{\mathcal{L}_1}(s) \text{ are combined into } \sigma(s)$$  \hfill (5)

The low-pass filter $C(s)$ (see Fig. 1) is considered to be strictly-proper with $C(0) = 1$, such that:

$$H(s) \triangleq \frac{A(s)M(s)}{C(s)A(s) + (1 - C(s))M(s)} \text{ is stable}, \quad \text{and the following } \mathcal{L}_1 \text{-norm condition is satisfied:}$$

$$\|G(s)\|_{\mathcal{L}_1} L < 1, \quad \text{where } G(s) \triangleq H(s)(1 - C(s)),$$  \hfill (8)

where $L$ is the Lipschitz constant defined in Assumption [1].

The reference model that will describe the repeatable behavior of the system controlled by an underlying $\mathcal{L}_1$ adaptive controller is proven to be bounded-input, bounded-output (BIBO) stable in [10] using the $\mathcal{L}_1$-norm condition. The solution of the $\mathcal{L}_1$-norm condition exists under the following assumptions:

Assumption 2 (Stability of $H(s)$). $H(s)$ is assumed to be stable for appropriately chosen low-pass filter $C(s)$ and first-order reference eigenvalue $-m < 0$.

The above assumption holds for cases when $A(s)$ can be stabilized by a proportional-integral controller, as demonstrated in [10]. The equations that describe the implementation of the $\mathcal{L}_1$ output feedback architecture are presented below:

Output Predictor: The output predictor is used within the $\mathcal{L}_1$ adaptive output feedback architecture to estimate the system output based on the reference model [5] as:

$$\dot{\hat{y}}_1(t) = -m\hat{y}_1(t) + m(u_{\mathcal{L}_1}(t) + \hat{\sigma}(t)), \quad \hat{y}_1(0) = 0$$

where $\hat{\sigma}(t)$ is the adaptive estimate of $\sigma(t)$. In the Laplace domain,

$$\hat{y}_1(s) = M(s)(u_{\mathcal{L}_1}(s) + \hat{\sigma}(s)).$$  \hfill (9)

Adaptation Law: The following adaptation law is used to update the adaptive estimate $\hat{\sigma}(t)$:

$$\dot{\hat{\sigma}}(t) = \Gamma \text{Proj}(\hat{\sigma}(t), -\hat{y}(t)), \quad \hat{\sigma}(0) = 0,$$  \hfill (10)

where $\hat{y}(t) \triangleq \hat{y}_1(t) - y_1(t), \Gamma \in \mathbb{R}^+$ is the adaptation rate subject to the lower bound specified in [10]. To ensure fast adaptation, the adaptation rate is set very large. The projection operator as defined in [10] guarantees that the estimated value of $\sigma(t)$ remains within a specified convex set.

Control Law: The control law is calculated by passing through a low-pass filter the difference between the desired trajectory signal $r_1(t)$ and the adaptive estimate $\hat{\sigma}(t)$ as:

$$u_{\mathcal{L}_1}(s) = C(s)(r_1(s) - \hat{\sigma}(s)).$$  \hfill (11)

This means that the $\mathcal{L}_1$ adaptive controller compensates only for the low-frequency uncertainties within $A(s)$ and $d_{\mathcal{L}_1}(s)$, i.e., the frequencies that the system is able to counteract. The low-pass filter attenuates the high frequency portion.

Performance Bounds: In this framework, the $\mathcal{L}_1$ adaptive controller is chosen as it provides performance bounds on the estimation and output errors. The closed-loop reference system for the $\mathcal{L}_1$ adaptive controller is described as follows:

$$y_{1, \text{ref}}(s) = M(s)(u_{\text{ref}}(s) + \sigma_{\text{ref}}(s)),$$  \hfill (12)

where

$$\sigma_{\text{ref}}(s) = \frac{(A(s) - M(s))u_{\text{ref}}(s) + A(s)d_{\text{ref}}(s)}{M(s)}.$$

and $d_{\text{ref}}(s)$ is the Laplace transform of $d_{\text{ref}}(t) \triangleq f(t, y_{1, \text{ref}}(t))$. If $C(s)$ and $M(s)$ satisfy the $\mathcal{L}_1$-norm condition [8], then the closed-loop reference system described in [12] and [13] is BIBO stable.

In [10], it is shown that when system (1) is subject to the $\mathcal{L}_1$ output feedback adaptive controller described in [9], [10], and [11], and $C(s)$, and $M(s)$ satisfy the $\mathcal{L}_1$-norm condition in [8], the following bounds hold:

$$\|\tilde{y}\|_{\mathcal{L}_1} < \gamma_0,$$  \hfill (14)

$$\|y_{1, \text{ref}} - y_1\|_{\mathcal{L}_1} < \gamma_1,$$  \hfill (15)

where $\tilde{y}(t) \triangleq \hat{y}_1(t) - y_1(t), \gamma_0$ is defined in equation (4.31) in [10] and

$$\gamma_1 \triangleq \left\| \frac{C(s)H(s)}{M(s)} \right\|_{\mathcal{L}_1} L \gamma_0.$$  

It is important to note that $\gamma_0 \propto \sqrt{\frac{1}{\tau}}$, which means that for high adaptation gains, the estimation error is bounded and can be made arbitrarily small. Furthermore, the actual system output approaches the behavior of the reference system [12].

When the uncertainties are within the bandwidth of the low-pass filter, the controller is able to cancel the uncertain-
ties exactly. In this ideal scenario, the system response is the following:
\[ y_{1,\text{id}} = M(s) r_1(s). \]  
(16)

Even when the ideal case is not achieved, the transient and steady-state performance of the controller have the performance guarantees provided in [14] and [15]. For the proposed framework, we require that the system controlled by the \( L_1 \) adaptive controller behaves as close as possible to a known, linear model. As shown in [15], in order to achieve this we need to choose \( \Gamma \) very large as it is inversely proportional to the bound \( \gamma_1 \).

1) Multi-Input, Multi-Output Implementation: The SISO architecture described above can be extended to a MIMO implementation. We assume that the states are decoupled, which can be achieved after applying an appropriate feedback linearization. Then, for \( n \) different outputs, the low-pass filter \( C(s) \) and the first-order output predictor [9] are implemented as \( (n \times n) \) diagonal transfer function matrices:
\[
C(s) = \text{diag}(C_1(s), \ldots, C_n(s)),
\]
\[
M(s) = \text{diag}(M_1(s), \ldots, M_n(s)),
\]  
(17)

where \( C_i(s) = \frac{\omega_i}{s + \omega_i}, \omega_i > 0, M_i(s) = \frac{m_i}{s + m_i}, m_i > 0, \) and \( i = 1, \ldots, n \). We further define \( K \) as the proportional gain:
\[
K = \text{diag}(K_1, \ldots, K_n),
\]
and \( K_i \in \mathbb{R}^+ \) for \( i = 1, \ldots, n \).

B. Model Predictive Control

MPC is used to find the optimal control input sequence that steers the dynamic system to a desired state, taking into account constraints in the values of the control inputs. This algorithm updates the control signal at every sampling time based on a known system model and desired future output. The key dynamics of the system to be controlled can be described by the following linear discrete-time model:
\[
y_1(k + 1) = A y_1(k) + B r_1(k), \quad y_1(0) = y_{1,0},
\]  
(18)

where \( y_1(k) \in \mathbb{R}^n \) is the output at time step \( k \), and \( r_1(k) \in \mathbb{R}^p \) is the control input. The desired output trajectory \( y_1^*(k) \) is assumed to be feasible based on the model [18], where \( (r_1^*(k), y_1^*(k)) \) satisfy [18]. The receding horizon implementation is formulated by introducing the following open-loop optimization problem:
\[
\min_{r_1(k), \ldots, r_1(k+N_h)} J(y_1(k), r_1(k))
\]  
(19)

subject to
\[
y_1(t + 1) = A y_1(t) + B r_1(t), \quad \forall t = k, \ldots, k + N_h
\]

where \( J(y_1(k), r_1(k)) \) is defined as:
\[
J(y_1(k), r_1(k)) = 
\sum_{t=k}^{k+N_h} (y_1^*(t) - y_1(t)) Q (y_1^*(t) - y_1(t)) + 
\sum_{t=k}^{k+N_h} r_1(t) R r_1(t) + \delta r_1(t) T S \delta r_1(t),
\]  
(20)

where \( \delta r_1(t) = r_1(t) - r_1(t-1), Q, R, S \in \mathbb{R}^{p \times p}, \) and \( N_h \) is the length of the prediction horizon.

C. Adaptive Model Predictive Control

The proposed architecture uses MPC with an underlying \( L_1 \) adaptive controller (see Fig. 1) which guarantees that the controlled system behaves close to a reference model. The MPC updates the input signal to the underlying \( L_1 \) adaptive controller \( r_1(k) \) at every sampling time based on a known system model and desired future output. We assume that the \( L_1 \) adaptive controller makes the system behave close to the system [19], which is achievable if the low-pass filter \( C(s) \) in [17] is selected to be compatible with the control channel specifications. In the ideal scenario, the system response is described by [16] where \( M(s) \) is defined as in [17] and can be discretized and written as:
\[
y_1(k + 1) = A_D y_1(k) + B_D (r_1(k) - y_1(k)),
\]  
(21)

where \( A_D \) and \( B_D \) are the minimal state-space realizations (?) of \( M(s) \). We can then reformulate the open-loop optimization problem as:
\[
\min_{r_1(k), \ldots, r_1(k+N_h)} 
\sum_{t=k}^{k+N_h} (y_1^*(t) - y_1(t)) Q (y_1^*(t) - y_1(t)) + 
\sum_{t=k}^{k+N_h} r_1(t) R r_1(t) + \delta r_1(t) T S \delta r_1(t)
\]  
(22)

subject to
\[
y_1(k + 1) = A_D y_1(k) + B_D (r_1(k) - y_1(k)),
\]
\[
|r_1(t)| \leq r_{\text{max}}, \quad \forall t = k, \ldots, k + N_h,
\]

where the second constraint may be added to ensure a smooth input and \( r_{\text{max}} \) is designed based on the system’s physical constraints.

IV. EXPERIMENTAL RESULTS

This section shows experimental results of the proposed framework combining \( L_1 \) adaptive control and MPC (MPC-L1) applied to a quadrotor flying three-dimensional trajectories when it is subject to dynamic disturbances. We assess two main aspects of the proposed framework: (i) predictive performance, and (ii) robustness to external disturbances, as compared to non-predictive and/or non-adaptive frameworks.

A. Quadrotor Setup

The SISO architecture presented in the previous section is extended to the MIMO quadrotor system. This is done by extending the low-pass filter and first-order reference system transfer functions into \( (3 \times 3) \) transfer function matrices.
with low-pass filter and first-order reference system transfer functions, respectively, in the diagonal.

We define $y_1(t)$ and $y_2(s)$ as the Laplace transforms of the translational velocity $y_1(t)$ and position $y_2(t)$. We also define $r_1(s)$ and $r_2(s)$ as the Laplace transforms of the desired translational velocity $r_1(t)$ and the desired bounded continuous position reference input $r_2(t)$, respectively. Each element of the three-dimensional signals and each element of the transfer function matrices correspond to the $x$, $y$ and $z$ directions.

In order to ensure that the quadrotor remains within certain position boundaries, we implement an outer-loop proportional controller [14], [19]. It is important to note that $y_2(s) = \frac{1}{s}y_1(s)$; hence, the input to the $L_1$ adaptive output feedback controller $r_1(i)$ depends on the output of the system $y_1$ through the feedback defined by:

$$r_1(i) = K(r_2(i) - y_2(i)).$$

This extension to the $L_1$ adaptive controller makes the system behave like the following linear reference system:

$$y_{2, id}(s) = \begin{bmatrix} D_1(s) & 0 & 0 \\ 0 & D_2(s) & 0 \\ 0 & 0 & D_3(s) \end{bmatrix} r_2(s),$$

(23)

where

$$D_i(s) = \frac{K_i m_i}{s^2 + m_i s + K_i m_i} \quad \text{for } i = 1, 2, 3.$$  (24)

The system in (23) is the system used as constraint in the MPC implementation [22]. We use the IBM CPLEX solver to compute the above MPC optimization problem.

B. Experimental Setup

The experiments were performed using the Parrot Bebop 2 quadrotor. An overhead motion capture camera system was used to obtain position, velocity, rotation and rotational velocity measurements. Let $r_{2,j}(t)$ be the desired translational positions and $y_{2,j}(t)$ be the measured quadrotor positions in the $j = x, y, z$ directions, respectively. We propose five different trajectories to test our approach, as shown in Fig. 2.

To quantify the controller performance, we define the average position error as:

$$e = \frac{1}{N} \sum_{i=1}^{N} \sqrt{e_x^2(i) + e_y^2(i) + e_z^2(i)},$$

(25)

where $e_j(i) = r_{2,j}(i) - y_{2,j}(i)$ and $j = x, y, z$.

C. $L_1$ Adaptive Control Performance Bounds

In this subsection we experimentally verify that (i) the estimation error $\hat{y}$ remains bounded, and (ii) the actual system remains close to the ideal linear system for the extended $L_1$ adaptive controller described by (23). To do this, we use reference input $r_2$ to command the drone to hover for 1.5 seconds after which it starts moving in a 3D straight line. Fig. 3 shows, on the left side, the estimation error $\hat{y}_{1,i} - y_{1,i}$ for each axis $i = x, y, z$. It is clear that for all three axes the estimation error remains bounded. Moreover, using the linear reference model (23), we calculate the trajectory $y_{2, id}$ that the ideal reference system would follow if the reference input $r_2$ was applied. The right side of Fig. 3 shows in red the trajectory of the ideal reference system $y_{2, id}$ and in blue the actual trajectory that the quadrotor followed $y_2$, when the input $r_2$ was applied. The actual system does behave close to the ideal reference system.

D. Predictive Performance

We first demonstrate the benefit of the predictive MPC component of the proposed approach. To do this, we compare the tracking performance obtained with the proposed MPC-$L_1$ to that obtained with a PID-$L_1$ and LQR-$L_1$ framework. The PID-$L_1$ framework uses a proportional-integral-derivative (PID) controller to modify the input $r_2$ of an underlying $L_1$ adaptive controller. The LQR-$L_1$ framework...
uses an infinite horizon linear-quadratic regulator (LQR) with integral feedback controller to modify the input $r_2$ of an underlying $L_1$ adaptive controller. The integral feedback in the LQR is used to decrease the steady-state error. The LQR is calculated based on the model $23$. It is important to note that neither the PID-$L_1$ nor the LQR-$L_1$ controllers take into account future desired outputs when computing the input to the underlying $L_1$ adaptive controller.

We compare the three controllers on the five test trajectories shown in Fig. 2. The average position tracking errors for each controller and trajectory are shown in Fig. 4. The PID-$L_1$, LQR-$L_1$ and MPC-$L_1$ frameworks to track each of the five desired trajectories: PID-$L_1$, LQR-$L_1$ and MPC-$L_1$ when tracking the purple trajectory in Fig. 2 are shown. The feedback-only controllers present a significant tracking delay as shown by the blue and black lines. The predictive component of the proposed MPC-$L_1$ framework computes the optimal input that minimizes the tracking error and achieves a better performance, as shown by the cyan line in Fig. 5. This shows the benefit of the predictive MPC component of the proposed approach.

E. Robustness to Disturbances

To assess the robustness to disturbances, the proposed framework is compared to an MPC-PID framework. The MPC-PID framework uses an MPC to modify the input to an underlying PID controller. The system model used in the MPC is obtained by applying a step input to $x$, $y$, and $z$ directions separately and characterizing the system response when the quadrotor is controlled by the PID controller. The system is assumed to be a second order system in each direction. We first use the MPC-PID and MPC-$L_1$ frameworks to track each of the five desired trajectories. The cost functions used in the MPC-PID and MPC-$L_1$ frameworks are the same.

The average position errors for the MPC-PID and MPC-$L_1$ approaches are shown in Fig. 6 in dark blue and dark red, respectively. The system model obtained through the step input response for the MPC-PID framework is not accurate enough to describe the system behavior. We found that tuning this system model for each trajectory could improve performance for that trajectory; however, doing so affected the performance of the other trajectories. In order to fairly compare both approaches, the model used in the MPC-PID framework obtained through applying a step input is kept constant across trajectories.

Next, using a fan we introduce a wind disturbance at different points in each trajectory. The resulting average errors when the fan disturbance is applied are shown in Fig. 6 in light blue and light red for the MPC-PID and MPC-$L_1$ frameworks, respectively. The proposed MPC-$L_1$ framework is able to keep approximately the same performance when a disturbance is applied since the underlying $L_1$ adaptive controller is able to compensate for it. In some cases the disturbance applied to the system may steer the system in the direction that reduces the tracking trajectory error as in trajectory 3, where the error in the MPC-PID approach decreases after the application of the wind disturbance. However, the proposed framework has, on average, a smaller increase in the error when the disturbance is applied.

V. CONCLUSIONS

In this paper, we introduce a novel adaptive MPC framework to improve trajectory tracking performance in the presence of disturbances and partially unknown dynamics. The framework relies on the $L_1$ adaptive controller that makes the system remain close to a predefined linear system behavior as specified by a reference model, despite the presence of disturbances and changing dynamics. The MPC is used to calculate an ‘optimal’ reference input, as specified by a quadratic cost function, to modify the desired input for the underlying $L_1$ adaptive controller. We validated the assumptions of the proposed approach through experiments on a quadrotor. Finally, we showed through experiments on a quadrotor that the proposed method improves the tracking performance of the system as compared to non-predictive approaches (PID-$L_1$ and LQR-$L_1$) and a non-adaptive approach (MPC-PID), even in the presence of strong wind disturbances. We hope the reader finds this an interesting approach for controlling nonlinear systems with an adaptive linear MPC approach, at a reduced computational cost.

REFERENCES

Fig. 5. Actual position of the quadrotor tracking the purple trajectory in Fig. 2 for each of the three controllers: PID-$L_1$, LQR-$L_1$, and MPC-$L_1$. The feedback-only controllers PID-$L_1$ and LQR-$L_1$ present tracking delay. The proposed MPC-$L_1$ framework reduces the tracking error as it is able to include future desired outputs in the calculation of the optimal input.

Fig. 6. Average position tracking errors over five different trajectories when using the MPC-PID and MPC-$L_1$ controllers. The cases when no disturbance is applied to the system are depicted in dark blue (MPC-PID) and dark red (MPC-$L_1$). A fan is used to apply a wind disturbance to the system and the resulting errors are depicted in light blue (MPC-PID) and light red (MPC-$L_1$). The proposed MPC-$L_1$ framework is able to compensate for the disturbance applied and has a performance similar to the case when no disturbance is applied.


