STOCHASTIC SHORTEST PATH

- system evolution

$$x_{k+1} = \omega_k, \qquad k = 0, 1, \dots$$

$$\begin{cases} x_k \in S = \{1, 2, \dots t\} \\ u_k \in C \\ \omega_k \in S \end{cases}$$

→ input constraints $\qquad u_k \in U(x_k) \subset C$

→ transition probabilities $\qquad p_{ij}(u) = P(\omega_k = j \mid x_k = i, u_k = u)$

$\qquad p_{tt}(u) = 1 \quad \forall u \in U(t)$

- objective

$$\min_{u_k \in U(x_k)} \lim_{N \to \infty} E\left(\sum_{k=0}^{N-1} g(x_k, u_k) \right)$$

with $g(t, u) = 0 \quad \forall u \in U(t)$

➡ infinite-horizon problem, iteration-independent, special cost-free termination state (Ass. 7.2.1)

"reach termination state with minimum expected cost"

BELLMAN'S EQUATION

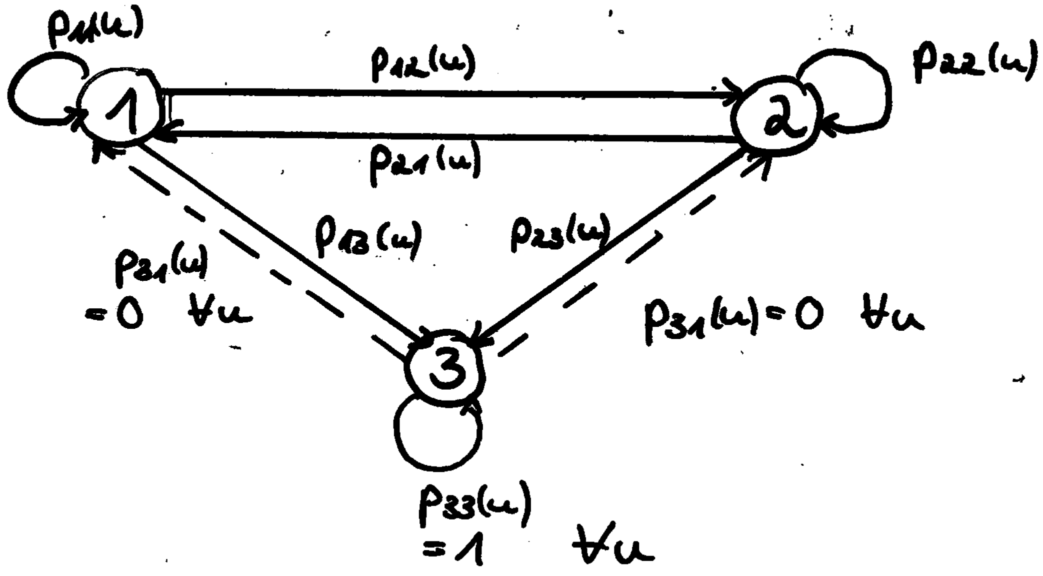
Optimal (infinite-horizon) cost from state i is <u>unique</u> solution of

$$J^*(i) = \min_{u \in U(i)} \left[g(i, u) + \sum_{j=1}^{n} p_{ij}(u) J^*(j) \right]$$

$$i = 1, 2, \dots n$$

$J^*(t) = 0 \qquad \Rightarrow$ obtain <u>stationary</u> optimal policy $\mu(i)$

STOCHASTIC SHORTEST PATH



⇒ termination state
⇒ finite number of nodes
⇒ transition probabilities

PERRON-FROBENIUS THEOREM

For $A \in \mathbb{R}^{n \times n}$ with strictly positive entries $a_{ij} > 0$, the following statement holds:

(a) $\exists!$ $\lambda_0 := \lambda(A)$ with $\underline{\lambda_0 > |\lambda_i(A)|}$, $\lambda_0 \in \mathbb{R}$

strictly larger than the absolute value of all other eigenvalues of A

(b) $\exists!$ $v \in \mathbb{R}^n$ with all entries strictly <u>positive</u> and

$$Av = \lambda_0 v .$$

(c) $\underline{\min_i \sum_j a_{ij} \leq \lambda_0 \leq \max_i \sum_j a_{ij}}$

(row sums)

$\underline{\min_j \sum_i a_{ij} \leq \lambda_0 \leq \max_j \sum_i a_{ij}}$

(column sums)