GENERAL PROBLEM FORMULATION

- system evolution

$$x_{k+1} = f_k(x_k, u_k, w_k), \qquad k = 0, 1, \dots N-1$$

$$\begin{cases} x_k \in S_k & \text{state} \\ u_k \in C_k & \text{control} \\ w_k \in D_k & \text{disturbance} \end{cases}, \qquad x_0 \text{ given}$$

→ input constraints $u_k \in U(x_k) \subset C_k$

→ probability distribution $w_k \sim P_k(\cdot \mid x_k, u_k)$

- objective

$$\min E\left\{ g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k) \right\}$$

DYNAMIC PROGRAMMING

Find an optimal policy $\pi^* = (\mu_0^*, \dots \mu_{N-1}^*), \; u_k^* = \mu_k^*(x_k)$

$$\begin{cases} J_N(x_N) = g_N(x_N) \\ J_k(x_k) = \min\limits_{u_k \in U_k(x_k)} \underset{w_k}{E} \left\{ g_k(x_k, u_k, w_k) + J_{k+1}(f_k(x_k, u_k, w_k)) \right\} \quad (*) \end{cases}$$

$\Rightarrow$ optimal cost: $J^*(x_0) = J_0(x_0)$

optimal policy: $u_k^* = \operatorname{argmin}(\dots)$